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第一部分 GMAT数学总论

第1章 考试简介

GMAT 数学是我们血统优良中国人的优势项目。据坊间传闻，在 GMAC 未发布的统计排名中，中国考生的数学平均分历年来独孤求败，视俄罗斯德国法国这些传统数学强国于无物。饶是如此，我们对数学的态度应该是战略上秒杀敌人，战术上蚕食对手。稍有不慎，反而会被数学分数拖了后腿。先简单介绍一下 GMAT 数学考察的内容。

1.1 GMAT数学的考查内容

据 OG 描述，GMAT 数学主要是在**算术**(Arithmetic)，**代数**(Algebra)，**几何**(Geometry)，和**应用题**(Word Problem)四个领域中考察学生的三种能力点：数学推理能力，数学解题能力和数学图形理解。

这里面唯一需要注意的是第一个：数学推理能力。对这种能力的考察是 GMAT 数学跟我们以往接触的数学考试略有不同的地方。而这种区别主要是通过第二种题型，也就是“数据充分性”体现出来的。下面给大家简单介绍题型及考试形式。

1.2 GMAT数学的考试形式

GMAT 数学部分共 21 道题，要求考生在 45 分钟内完成。只有一种题型：问题求解 (Problem Solving，简称 PS)。

在 Data Insights 模块中的数据充分性 (Data Sufficiency，简称 DS) 题型与数学部分有较高协同性，所以本资料将 PS 与 DS 集合在一起。

前一种题型与我们熟悉的传统数学考试选择题毫无二致，除了选项是 ABCDE 五个而不是四个以外。需要注意的是第二种题型：数据充分性也就是 DS 题型。这是 GMAT 原创的一种独特题型，下面给大家举一个例子：

If a real estate agent received a commission of 6 percent of the selling price of a certain house, what was the selling price of the house?

(1) The selling price minus the real estate agent's commission was \$84,600.

(2) The selling price was 250 percent of the original purchase price of \$36,000.

(A) Statement (1) ALONE is sufficient, but statement (2) is not sufficient.

(B) Statement (2) ALONE is sufficient, but statement (1) is not sufficient.

(C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

(D) EACH statement ALONE is sufficient.

(E) Statements (1) and (2) TOGETHER are NOT sufficient.

答案选 D。

它的主要特点是：先通过题面描述问题，然后问当以下两个条件（1）和（2）分别成立或者共同成立时，能否求解。五个选项是固定设计的，分别是

- (A) 只有条件（1）成立时能够独立求解；只有条件（2）成立时不能够独立求解。
- (B) 只有条件（2）成立时能够独立求解；只有条件（1）成立时不能够独立求解。
- (C) 条件（1）和条件（2）同时成立时能够求解，单独成立时不能够求解。
- (D) 条件（1）和条件（2）分别单独成立时都能够求解。
- (E) 条件（1）和条件（2）同时成立时也不能求解。

1.3 GMAT数学的计分情况

数学分值区间为60-90。中国考生数学平均分全球第一，应该在85和90之间。所以我们的目标也就是这个分数，意味着：21道数学题，我们最多可以错3个以内。

1.4 GMAT数学的难度

前面已经说过，GMAT 数学对于我国考生不构成结构性威胁。有一种常见的说法，就是把所有数学题翻译成中文，基本上属于我国中考数学的难度，知识点不超出初中范围。因此数学的主要难度就分布在英文理解上。这一说法应该说不够准确。

首先，从知识点方面来说（我们后面会给大家逐一介绍），大部分的算术、代数和几何确实相当于我们初中数学的知识点范围。但是也会有少量题目涉及到排列组合，概率论和统计学等我们国家高中甚至大学才会学到的知识点。

其次，Data Insights 数据充分题型，也就是DS 题型，需要一种严密周全的思维方式，这种思维方式是我们在中学数学中很少涉及到的，这里面包含了一些考察我们思维漏洞的出题套路，一不留神就容易做错。但是就GMAC 小老头儿们那点儿水平，也弄不出什么惊天地泣鬼神的花样儿来，再炫的霹雳组合拳在我们中国人看来也不过是花拳绣腿，经过短期的系统训练，帝国主义必然只是纸老虎。

第三点，就是一些应用题，如利率，收益和集合等。这类题目或者触及到我们英文理解的软肋，或者让某些考生感觉到无从下手，难以转化成数学问题。我们后面会给大家详细剖析个中缘由。

综上所述，GMAT 数学即使翻译成中文，难度应该也略超中考难度，介于中考高考之间。但是我们应该认识到，这种难度不是建立在计算量和复杂程度上的，而是建立在思维的严密度上的。因此，即使是那些数学基础薄弱的文科考生，只要思维跟上我们的授课体系，一两个月内将数学部分拿下完全没有问题。

第2章 本书的结构

2.1 五大错误原因

云帆美研的GMAT 资深名师根据自己的多年的经验总结,将中国考生在数学方面面对的主要困难分为以下五大类:

- ◎ 知识点遗忘
- ◎ 理解错误
- ◎ 粗心马虎
- ◎ 思维陷阱
- ◎ 疑难杂症

2.1.1 知识点遗忘和理解错误

在五大难题中,尤以**知识点遗忘**和**理解错误**最为猖獗,构成 GMAT 数学 50 分达标跑道上最大的两只拦路猛虎。

后者,也就是**理解错误**可以说是不言自明,英文的数学表达毕竟跟我们的习惯方式有着语言上的较大区别,英文理解的障碍、对数学术语的不熟悉还有对图表形式的不适应都会增加我们的审题难度。在延缓做题速度的同时,也有可能因为误读而将整个题目完全理解错误。

而前者,也就是**知识点遗忘**的情况则因人而异,数学方面比较扎实的考生,距离高考时间不是特别久远的考生,理工类考生,和记忆力超群的考生也许对这些知识点一见如故,完全没有陌生感;而数学功底偏弱的考生,文科类考生,工作多年很少接触数学的考生,还有记忆力不太好的考生也许会

跟这些知识点形同陌路,不禁感叹已将曾经朝夕相伴的数学老友忘掉了九霄云外。

在后面的**知识点遗忘**章节中,本书将根据 GMAC 明确规定的数学考察范围,也就是算术(arithmetic)、代数(algebra)和几何(geometry)三大科目进行逐个知识点详细讲解。在上一章提及的排列组合,统计和概率等较难知识点将会安排在算术的后半部分中,方便大家根据自己的情况灵活安排阅读的进度。

应用题(word problems)部分的讲解我们会放在**理解错误**中,因为我们中国考生经常发生的理解障碍总是在应用题型中感受的尤为明显,特别是投资问题、利率问题和工作效率等跟经济学有密切关系的语言点,我们将会结合真题给大家剖析常见的错误缘由。

2.1.2 粗心马虎、思维陷阱和疑难杂症

除了以上两大最主要原因以外,中国考生还经常在以下三个方面犯错而造成失分:**粗心马虎**,**思维陷阱**和**疑难杂症**。对这三种错误的研究和总结凝结着博森众位数学老师多年来的实践与教研结晶,也体现了 GMAT 作为一种独特的思维能力考试,力图在思维严密度方面对未来商科人才进行遴选鉴别的考察标准。

先说**粗心马虎**，尽管这是一个非常个性化的问题，但从统计学上讲，它还是有很多共性可以总结。常见的马虎问题包括单位的换算错误、计算失误、答非所问及统计元素个数的不正确等。我们将逐个攻破之，瓦解之，毁尸灭迹之。

然后是**思维陷阱**。如第 1 章所述，中国考生的共同特点是计算能力相对较强，而逻辑思维相对较弱，造成 GMAT 考试中 DS 题型的得分偏低。本书总结出 GMAT 数学考试常设的陷阱迷局，帮你排雷的同时，也试图助你培养出一种审慎严谨的思维方式，为今后在商科学习过程中面对的风暴式头脑训练打下预热准备。

此外，考生经常反映在能够读懂题目的情况下无从下手，知道属于哪一部分的考点却不能轻松得分。这一类问题我们总结为最后一点，也就是**疑难杂症**。在这部分中，我们还将传授一些临场做题的解答技巧，加快做题速度的非常规招式，参透题目内涵的思维方法，还有用来检查自己答案是否正确的查证手段，不光是解决疑难杂症的灵丹妙药，也可做突破 51 分极限的独门偏方。

2.2 本书章节

针对五个错误点，我们将会用五个章节：（第 3 章）知识点的遗忘、（第 4 章）理解错误、（第 5 章）马虎问题、（第 6 章）思维陷阱和（第 7 章）疑难杂症，一一给大家展开讲解。不光结合不同题型，考点类型，结合真题讲述，还会给大家提出克服错误的方法建议。

第三部分中，我们将提供一些数学习题给大家使用，其中一些重点题目的解题过程将会在光盘里选讲。

在第四部分的附录内容里，我们会给大家提供 GMAT 数学常考术语和公式，以及度量转换的列表，方便大家在考试前突击使用。

第二部分 五大错误原因逐个攻破

第3章 知识点的遗忘

许多 GMAT 的考生都参加工作很多年了，已经把原有的数学知识点忘得差不多了。本章中，我们将全面系统地归纳、讲解 GMAT 数学考点，免去考生重读教科书之苦。

3.1 算术基础

3.1.1 数的概念

整数(Integer)

集合 $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ 中的任意数字。

奇数(Odd Integer)

不能被 2 所整除的整数，如 1, -1, 3, -3, ……

偶数(Even Integer)

能够被 2 所整除的整数，如 0, 2, -2, 4, -4, ……注意 0 是偶数。

质数(Prime Numbers)

除了 1 和它本身之外，不能被其他正整数所整除的自然数，如 2, 3, 5, 7, 11, ……其中 2 是最小的质数。

合数(Composite Numbers)

除了 1 和它本身之外，还有其他因子的自然数，如 4, 6, 8, 9, 10, ……其中 4 是最小的合数(注：质数和合数都不能为负数，0 和 1 既不是质数也不是合数)。

互质数(Mutual Prime Numbers)

如果两个数的最大公约数为 1，那么这两个数叫做互质数。例如：13 和 15, 19 和 23 等。

倍数和可整除(Multiple and Divisible)

当整数 a 能被另一个整数 b 所整除(divisible)时， a 称为 b 的倍数(multiple)， b 称为 a 的约数或因数(divisor)。例如：10 是 5 的倍数，5 是 10 的约数。

公倍数(Common Multiple)

如果一个数同时是几个数的倍数，则称这个数为它们的公倍数；公倍数中最小的称为最小公倍数(lowest common multiple)。例如：12、24、36 等都是 2、4、6、12 的公倍数，其中 12 是它们的最小公倍数。

公约数或公因数(Common Factor or Divisor)

如果一个数同时是几个数的约数，则称这个数为它们的公约数或公因数；公约数中最大的被称为最大公约数(公因数)(greatest common factor or divisor)。例如：2、7、14 都是 28、42、70 的公约数，14 是它们的最大公约数。

完全平方数(Perfect Square)

若一个整数开平方后还是整数,则这个数被称之为完全平方数。例如: 4, 9, 16, 25, ……完全平方数均为自然数。

完全立方数(Perfect Cube)

若一个整数开三次方后还是整数,则这个数称之为完全立方数。例如: -27, -8, 0, 8, 27, ……

商和余数(Quotients and Remainders)

当一个正整数除以另一个正整数其商不为整数时就存在商和余数问题。余数和商为大于或等于零的整数,余数总小于除数。例如: 15 除以 7 时,其商为 2,余数为 1。

连续整数(Consecutive Integers)

按从小到大的顺序相连的几个整数称为“连续整数”。例如: -2、-1、0、1、2 是五个连续的整数。连续正整数的算术平均值也是首项和末项的算术平均值。此外, GMAT 还会涉及到连续偶数数列 (consecutive even integers) 和连续奇数数列 (consecutive odd integers) 的概念。

有理数(Rational Number)

能够写成两个整数相除形式的数。例如: 2 、 $\frac{2}{3}$ 、 $\frac{15}{31}$ 。

无理数(Irrational Number)

不能够写成两个整数相除形式的数。例如: $\sqrt{2}$ 、 $2+\sqrt{3}$ 。

3.1.2 数的性质

自然数、整数、奇数、偶数、质数、合数等有以下性质:

1. 奇数个奇数相加减,其结果必为奇数。

$$\text{例: } 1-3+5=3 \quad 1+3+5=9 \text{ 是奇数}$$

2. 偶数个奇数相加减,其结果必为偶数。

$$\text{例: } 1-3+5-7=-4$$

$$1-3+5+7=10$$

3. 奇数和偶数相加减,其结果必为奇数。

$$\text{例: } 1+2=3$$

$$1-2=-1$$

4. 任意多个偶数相加减,其结果仍为偶数。

$$\text{例: } 2+4+6-8=4$$

5. 若 n (n 为 >1 的自然数) 个整数连乘其结果为奇数,则这 n 个整数必然都是奇数。

$$\text{例: } 1 \times 3 \times 5 \times 7 = 105$$

$$-3 \times 1 \times 5 = -15$$

6. 若 n (n 为 >1 的自然数) 个整数连乘其结果为偶数,则这 n 个整数中至少有一个为偶数。

$$\text{例: } 2 \times 1 \times 3 \times 5 = 30$$

$$2 \times 4 \times 8 \times 1 \times 3 = 192$$

7. 若 n (n 为 >1 的自然数) 个连续的整数相加等于 0, 则 n 必为奇数。

例:

$$\underline{(-3)+(-2)+(-1)+0+1+2+3=0}$$

7 个数

8. 若 n (n 为 >1 的自然数) 个连续的奇数相加等于 0, 则 n 必为偶数。

例: $\frac{(-3)+(-1)+1+3}{4} = 0$
4 个数

9. 若 n (n 为 >1 的自然数) 个连续的偶数相加等于 0, 则 n 必为奇数。

例: $\frac{(-4)+(-2)+0+2+4}{5} = 0$
5 个数

10. 自然数间相加或相乘必然还为自然数。

11. 自然数间相减必然为整数(有可能为正, 有可能为负)。

12. 奇数个连续的整数的算术平均值等于这奇数个数中中间大小那个数的值。

例: $\frac{1+2+3+4+5}{5} = 3$

3 为 1、2、3、4、5 中中间大小的数。

13. 偶数个连续的整数的算术平均值等于这偶数个数中中间两个数的算术平均值。

例: $\frac{1+2+3+4+5+6}{6} = 3.5 = \frac{3+4}{2}$

3 和 4 为 1、2、3、4、5、6 中中间的两个数。

14. 奇数个连续的奇数的算术平均值等于这奇数个数中中间大小的那个数的值。

例: $\frac{(-3)+(-1)+1}{3} = -1$

-1 为 (-3)、(-1)、1 这 3 个数中中间大小的数。

15. 奇数个连续的偶数的算术平均值, 等于这奇数个数中中间大小的那个数的值。

16. 偶数个连续的奇数的算术平均值,

等于这偶数个数中中间两个数的算术平均值。

例: $\frac{1+3+5+7}{4} = 4 = \frac{3+5}{2}$

3 和 5 为 1、3、5、7 中中间的两个数。

17. 偶数个连续的偶数的算术平均值等于这偶数个数中中间两个数的算术平均值。

18. 任何一个大于 2 的偶数都可以表示为两个质数的和(注: 此为一未被证明的定理——歌德巴赫猜想, 但在考试中可应用上)。

例: $4 = 2+2$

$6 = 3+3$

$8 = 3+5$

.....

19. 2 个连续的自然数相乘必然为偶数。

例: $2 \times 3 = 6$

20. 3 个连续的自然数相乘必然为 6 的倍数。

例: $2 \times 3 \times 4 = 24$

21. 若三个连续的自然数的算术平均值为奇数, 则这三个自然数的乘积必为 24 的倍数(即两个连续的偶数相乘为 24 的倍数)。

例: $\frac{4+5+6}{3} = 5$

则 $4 \times 5 \times 6 = 120$ 可以整除 24。

例 1: The product of two consecutive positive integers CANNOT be

(A) a prime number

(B) divisible by 11

(C) a multiple of 13

(D) an even number less than 10

(E) a number having 4 as its units digit

翻译: 两个连续正整数的积不能是:

解答: 对于此类数字规律题应当找反例, 采用排除法。(A)不对, 因为惟一的例外是 $1 \times 2 = 2$ 。(B)显然不对, 如 11、12 等等。(C)同理不对, 如 13、14。(D) 是一个偶数是肯定的, 但完全可以大于 10, 所以不对。(E)找不到反例, 两个连续正整数的积个位数不可能为 4。所以(E) is correct。

例 2: If x and y are positive integers and x is a multiple of y , is $y = 2$?

(1) $y \neq 1$

(2) $x + 2$ is a multiple of y

翻译: 假如 x, y 是正整数并且 x 是 y 的倍数, $y = 2$?

(1) $y \neq 1$

(2) $x + 2$ 是 y 的倍数

解答: 由(1)中 $y \neq 1$ 说明不了 y 是否等于 2;

(2) $x + 2$ 是 y 的倍数, 且 x 是 y 的倍数, 则 $y = 1$ 或 2, 也无法单独回答上面的问题; 只有(1)和(2)条件都成立, 才可以得到 $y = 2$ 。

所以(C) is correct。

3.1.3 最大公约数和最小公倍数

1. 公约数和最大公约数

几个数公有的约数, 叫做这几个数的公约数, 其中最大的一个, 叫做这几个数的最大公约数。

例如: 12 的约数有 1、2、3、4、6、12;

18 的约数有 1、2、3、6、9、18。12 和 18 的公约数有 1、2、3、6, 其中 6 是 12 和 18 的最大公约数, 记作 $(12, 18) = 6$ 。

2. 公倍数和最小公倍数

几个数公有的倍数, 叫做这几个数的公倍数; 其中最小的一个, 叫做这几个数的最小公倍数。

例如: 12 的倍数有 12、24、36、48、

60、72、84, ……

18 的倍数有 18、36、54、72、90, ……

12 和 18 的公倍数有 36、72, ……其中 36 是 12 和 18 的最小公倍数, 记作 $[12, 18] = 36$ 。

3. 互质数

如果两个数的最大公约数是 1, 那么这两个数叫做互质数。

4. 最大公约数和最小公倍数的关系

定理 1 两个自然数分别除以它们的最大公约数, 所得的商互质, 即如果 $(a, b) = d$, 那么 $(a \div d, b \div d) = 1$ 。

定理 2 两个数的最小公倍数与最大公约数的乘积等于这两个数的乘积。

定理 3 两个数的公约数一定是这两个数的最大公约数的约数。

例 1: 加工某种机器零件, 要经过三道工序, 第一道工序每个工人每小时可完成 3 个零件, 第二道工序每个工人每小时可完成 10 个, 第三道工序每个工人每小时可完成 5 个, 要使加工生产均衡, 三道工序至少各分配几个工人?

(A) 3, 10, 5

(B) 10, 3, 5

(C) 5, 10, 3

(D) 10, 3, 6

(E) 10, 5, 3

解答: 因为 $[3, 10, 5] = 30$

所以各项工序均应加工 30 个零件。

$30 \div 3 = 10$ (人)

$30 \div 10 = 3$ (人)

$30 \div 5 = 6$ (人)

因此第一道工序至少要分配 10 人, 第二道工序至少要分配 3 人, 第三道工序至少要分配 6 人。

所以(D)is correct.

提示: 要使加工生产均衡, 各道工序生产的零件总数应是 3、10 和 5 的公倍数, 要求三道工序“至少”要多少工人, 要先求 3、10 和 5 的最小公倍数。

例 2: 一次会餐供有三种饮料, 餐后统计, 三种饮料共用了 65 瓶; 平均每 2 个人饮用一瓶 A 饮料, 每 3 人饮用一瓶 B 饮料, 每 4 人饮用一瓶 C 饮料, 问参加会餐的人数是多少人?

- (A) 12
- (B) 24
- (C) 36
- (D) 48
- (E) 60

解答: 因为 $[2, 3, 4] = 12$

所以参加会餐人数应是 12 的倍数。

又因为 $12 \div 2 + 12 \div 3 + 12 \div 4$
 $= 6 + 4 + 3 = 13$ (瓶)

所以可见 12 个人要用 6 瓶 A 饮料, 4 瓶 B 饮料, 3 瓶 C 饮料, 共用 13 瓶饮料。

又因为 $65 \div 13 = 5$, 所以参加会餐的总人数应是 12 的 5 倍。

$$12 \times 5 = 60(\text{人})$$

所以(E)is correct.

提示: 由题意可知, 参加会餐人数应是 2、3、4 的公倍数。

例 3: 一张长方形纸, 长 2703 厘米, 宽 1113 厘米, 要把它截成若干个同样大小的正方形, 纸张不能有剩余且正方形的边长要尽可能大。问: 这样的正方形的边长是多少厘米?

解答: 由题意可知, 正方形的边长即是 2703 和 1113 的最大公约数, 在学校, 我们

已经学过用短除法求两个数的最大公约数, 但有时会遇到类似此题情况, 两个数除了 1 以外的公约数一下不好找到, 但又不能轻易断定它们是互质数, 怎么办? 在此, 我们以例 3 为例介绍另一种求最大公约数的方法。我们用大数去除以小数, 得到商和余数; 然后再用除数(即刚才的小数)去除以余数, 以此类推, 直至余数为 0。当余数为 0 时, 最后一个算式中的除数就是原来两个数(即大数和小数)的最大公约数。下面以此题为例具体介绍一下这种方法:

$$2703 = 2 \times 1113 + 477$$

$$1113 = 2 \times 477 + 159$$

$$477 = 3 \times 159$$

当余数为 0 时, 最后一个算式中的除数 159 就是原来两个数 2703 和 1113 的最大公约数。

可见, $477 = 159 \times 3$

$$1113 = 159 \times 3 \times 2 + 159 = 159 \times 7$$

$$2703 = 159 \times 7 \times 2 + 477$$

$$= 159 \times 7 \times 2 + 159 \times 3$$

$$= 159 \times 17$$

又因为 7 和 17 是互质数, 所以 159 是 2703 和 1113 的最大公约数。

提示: 我们把这种求最大公约数的方法叫做辗转相除法。辗转相除法的优点在于: 它能在较短的时间内求出任意两个数的最大公约数。

例 4: 若 r 和 s 为大于 1 的自然数, 且有 $11(s-1) = 13(r-1)$, $s+r$ 的最小值?

- (A) 24
- (B) 25
- (C) 26
- (D) 27
- (E) 28

解答: 既然有 $11(s-1) = 13(r-1)$ 则有 $11(s-1)$ 为 13 的倍数, 也即 13 是 $11(s-1)$ 的因子, 由于 11 和 13 最大公约数为 1, 因此 13 是 $s-1$ 的因子。既然取 $s+r$ 的最小值, 则尽量取 s 最小, $s-1$ 即是 13 的 1 倍, $s-1=13$ 求得 $s=14$, 同理 $r-1=11$, 即 $r=12$, 因而 $s+r$ 最小值为 26。所以(C) is correct。

提示: 若两数 a 和 b 的最大公约数为 1, 则称 a 和 b 互质。

例 5: 已知两数的最大公约数是 21, 最小公倍数是 126, 求这两个数的和是多少?

- (A) 105
- (B) 147
- (C) 150
- (D) 105 或 147
- (E) 105 或 150

解答: 要求这两个数的和, 我们可先求出这两个数各是多少, 设这两个数为 a, b , $a < b$ 。

因为这两个数的最大公约数是 21, 故设 $a = 21a_1$, $b = 21b_1$, 且 $(a_1, b_1) = 1$ 。因为这两个数的最小公倍数是 126。

所以 $126 = 21 \times a_1 \times b_1$,

于是 $a_1 \times b_1 = 6$,

$$\text{解出 } \begin{cases} a_1 = 1 \\ b_1 = 6 \end{cases}, \begin{cases} a_1 = 2 \\ b_1 = 3 \end{cases}。$$

$$\text{则 } \begin{cases} a = 21 \times 1 = 21 \\ b = 21 \times 6 = 126 \end{cases},$$

$$\begin{cases} a = 21 \times 2 = 42 \\ b = 21 \times 3 = 63 \end{cases}。$$

因此, 这两个数的和为 $21+126 = 147$ 或 $42+63 = 105$ 。

所以(D) is correct。

例 6: What is the greatest common divisor of positive integers m and n ?

(1) m is a prime number.

(2) m and n are consecutive integers.

翻译: 正整数 m 和 n 的最大公约数是多少?

(1) m 是一个质数。

(2) m 和 n 是连续整数。

解答: 由(1)无法判定 m, n 的最大公约数, 因为 n 完全是可能是 m 的倍数; (2) m, n 为连续正整数则必然互质, 可以得到最大公约数为 1。

所以(B) is correct。

提示: 连续正整数则必然互质, 其最大公约数为 1。考生不要忽略了这一性质。

例 7: For which of the following pairs of integers is the least common multiple of the integers minus their greatest common divisor the greatest?

- (A) 3, 12
- (B) 5, 6
- (C) 10, 20
- (D) 11, 12
- (E) 15, 30

翻译: 下列哪一对数的最小公倍数减最大公约数最大?

解答: the least common multiple 最小公倍数; the greatest common divisor 最大公约数

- (A) $12 - 3 = 9$
- (B) $30 - 1 = 29$
- (C) $20 - 10 = 10$
- (D) $132 - 1 = 131$
- (E) $30 - 15 = 15$

所以(D) is correct。

3.1.4 数的整除

1. 数的整除性质

如果 a 能被 b 整除, 记作 $b \mid a$ 。

性质 1: 如果 a 、 b 都能被 c 整除, 那么它们的和与差也能被 c 整除。

即: 如果 $c \mid a, c \mid b$, 那么 $c \mid (a \pm b)$ 。

性质 2: 如果 b 与 c 的积能整除 a , 那么 b 与 c 都能整除 a 。

即: 如果 $bc \mid a$, 那么 $b \mid a, c \mid a$ 。

性质 3: 如果 b 、 c 都能整除 a , 且 b 和 c 互质, 那么 b 与 c 的积能整除 a 。

即: 如果 $b \mid a, c \mid a$, 且 $(b, c)=1$, 那么 $bc \mid a$ 。

性质 4: 如果 c 能整除 b , b 能整除 a , 那么 c 能整除 a 。

即: 如果 $c \mid b, b \mid a$, 那么 $c \mid a$ 。

2. 数的整除特征

(1) 能被 2 整除的数的特征: 个位数字是 0、2、4、6、8 的整数。“特征”包含两方面的意义: 一方面, 个位数字是 0 或偶数的整数, 必能被 2 整除; 另一方面, 能被 2 整除的数, 其个位数字只能是 0 或偶数, 下面(2)~(7)的“特征”含义相似。

(2) 能被 5 整除的数的特征: 个位是 0 或 5。

(3) 能被 3 或 9 整除的数的特征: 各个数位数字之和能被 3 或 9 整除。

(4) 能被 4 或 25 整除的数的特征: 末两位数能被 4 或 25 整除。

(5) 能被 8 或 125 整除的数的特征: 末三位数能被 8 或 125 整除。

(6) 能被 11 整除的数的特征: 这个整数的奇数位上的数字之和与偶数位上的数字之和的差(大减小)是 11 的倍数。

例 1: 李老师为学校一共买了 28 支价格相同的钢笔, 共付人民币 $9\square.2\square$ 元, 已知 \square 处数字相同, 请问每支钢笔多少元?

(A) 3.51

(B) 3.61

(C) 3.71

(D) 3.81

(E) 3.91

解答: 因为 $9\square.2\square=9\square2\square$ 分

$$28 = 4 \times 7,$$

所以根据整除“性质 2”可知

4 和 7 均能整除 $9\square2\square$ 。

$4 \mid 2\square$, 可知 \square 处只能填 0 或 4 或 8。

因为 7 不能整除 9020、9424, 所以 \square 处不能填 0 和 4;

为 79828, 所以 \square 处应该填 8。

又因为 $9828 \text{ 分} = 98.28 \text{ 元}$

$$98.28 \div 28 = 3.51 \text{ 元}$$

因此每只钢笔 3.51 元。

所以(A) is correct。

例 2: 在 865 后面补上三个数字, 组成一个六位数, 使它能分别被 3、4、5 整除, 且使这个数值尽可能的小。问最小值是多少?

(A) 865000

(B) 865020

(C) 865025

(D) 865040

(E) 865050

解答: 设补上数字后的六位数是 $\overline{865abc}$,

因为这个六位数能分别被 3、4、5 整除, 所以它应满足以下三个条件:

第一, 数字和 $(8+6+5+a+b+c)$ 是 3 的倍数。

第二, 末两位数字组成的两位数 \overline{bc} 是 4 的倍数。

第三, 末位数字 c 是 0 或 5。

根据题意可知: 4 能整除 \overline{bc} , 且 c 只能取 0 或 5。

因为能被 4 整除的数的个位数不可能是 5, 所以 c 只能取 0, 因而 b 只能取自 0, 2, 4, 6, 8 中之一。

又因为 3 能整除 $\overline{865abc}$, 且 $(8+6+5)$ 除以 3 余 1, 所以 $a+b$ 除以 3 余 2。

为满足题意“数值尽可能小”, 只需取 $a=0$, $b=2$ 。所以要求的六位数是 865020。

所以(B)is correct。

3.1.5 同余的概念和性质

本节我们通过一个例题来引出同余的概念。

例 1: 1993 年的元旦是星期五, 1994 年的元旦是星期几?

这个问题也难不倒我们, 因为, 1993 年有 365 天, 而 $365=7 \times 52+1$, 所以 1994 年的元旦应该是星期六。

问题的实质是求用 7 去除某一总的天数后所得的余数, 在日常生活中, 时常要注意两个整数用某一固定的自然数去除, 所得的余数问题, 这样就产生了“同余”的概念, 如问题中的 15 与 365 除以 7 后, 余数是 1, 那么我们就说 15 与 365 对于模 7 同余。

同余定义: 若两个整数 a 、 b 被自然数 m 除有相同的余数, 那么称 a 、 b 对于模 m 同余, 用式子表示为:

$$a \equiv b \pmod{m}. \quad (*)$$

上式可读作:

a 同余于 b , 模 m 。

同余式 $(*)$ 意味着(我们假设 $a \geq b$):

$$a-b = mk, \quad k \text{ 是整数, 即 } m|(a-b).$$

同余式有以下性质: (其中 a 、 b 、 c 、 d 都是整数, m 是自然数)

性质 1: $a \equiv a \pmod{m}$ 。(反身性)

这个性质很显然, 因为 $a-a=0=m \times 0$ 。

性质 2: 若 $a \equiv b \pmod{m}$, 那么 $b \equiv a \pmod{m}$ 。(对称性)

性质 3: 若 $a \equiv b \pmod{m}$, $b \equiv c \pmod{m}$, 那么 $a \equiv c \pmod{m}$ 。(传递性)

性质 4: 若 $a \equiv b \pmod{m}$, $c \equiv d \pmod{m}$, 那么 $a \pm c \equiv b \pm d \pmod{m}$ 。(可加减性)

性质 5: 若 $a \equiv b \pmod{m}$, $c \equiv d \pmod{m}$, 那么 $ac \equiv bd \pmod{m}$ 。(可乘性)

性质 6: 若 $a \equiv b \pmod{m}$, 那么 $a^n \equiv b^n \pmod{m}$, (其中 n 为自然数)。

性质 7: 若 $ac \equiv bc \pmod{m}$, $(c, m)=1$, 那么 $a \equiv b \pmod{m}$, (记号 (c, m) 表示 c 与 m 的最大公约数)。

注意同余式性质 7 的 $(c, m)=1$, 否则像普通等式一样, 两边约去, 就是错的。

例如 $6 \equiv 10 \pmod{4}$, 而 $3 \not\equiv 5 \pmod{4}$, 因为 $(2, 4) \neq 1$ 。

例 2: 求 143^{89} 除以 7 的余数。

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

解答: 方法 1: 因为 $143 \equiv 3 \pmod{7}$

$$\text{所以 } 143^{89} \equiv 3^{89} \pmod{7}$$

$$\text{因为 } 89 = 64 + 16 + 8 + 1$$

$$\text{而 } 3^2 \equiv 2 \pmod{7},$$

$$3^4 \equiv 4 \pmod{7},$$

$$3^8 \equiv 16 \equiv 2 \pmod{7},$$

$$3^{16} \equiv 4 \pmod{7},$$

$$3^{32} \equiv 16 \equiv 2 \pmod{7},$$

$$3^{64} \equiv 4 \pmod{7}.$$

因为

$$\begin{aligned} 3^{89} &\equiv 3^{64} \times 3^{16} \times 3^8 \times 3 \\ &\equiv 4 \times 4 \times 2 \times 3 \\ &\equiv 5 \pmod{7}, \end{aligned}$$

所以 $143^{89} \equiv 5 \pmod{7}$ 。

因此 143^{89} 除以 7 的余数是 5。

方法 2: 证得 $143^{89} \equiv 3^{89} \pmod{7}$ 后,

$$3^6 \equiv 3^2 \times 3^4 \equiv 2 \times 4 \equiv 1 \pmod{7},$$

所以 $3^{89} \equiv (3^6)^{14} \equiv 1 \pmod{7}$ 。

$$\begin{aligned} 3^{89} &\equiv 3^{84} \times 3^4 \times 3 \\ &\equiv 1 \times 4 \times 3 \\ &\equiv 5 \pmod{7}. \end{aligned}$$

$143^{89} \equiv 5 \pmod{7}$ 。

所以(D) is correct.

提示: 同余的性质能使“大数化小”，凡求大数的余数问题首先考虑用同余的性质化大为小，这道题先把底数在同余意义下变小，然后从低次幂入手，重复平方，找找有什么规律。

例 3: 求自然数 $2^{100} + 3^{101} + 4^{102}$ 的个位数字。

- (A) 2
- (B) 3
- (C) 4
- (D) 5
- (E) 6

解答: 因为 $2^{100} \equiv 2^{4 \times 25} \equiv 6^{25} \equiv 6 \pmod{10}$,

$$3^{101} \equiv 3^{4 \times 25} \times 3^1 \equiv 1^{25} \times 3^1 \equiv 3 \pmod{10},$$

$$4^{102} \equiv 4^2 \equiv 6 \pmod{10},$$

所以 $2^{100} + 3^{101} + 4^{102} \equiv 6 + 3 + 6 \equiv 5 \pmod{10}$,

即自然数 $2^{100} + 3^{101} + 4^{102}$ 的个位数字是 5。

提示: 求自然数的个位数字即是求这个自然数除以 10 的余数问题。

3.1.6 质数和合数

1. 质数与合数

一个数除了 1 和它本身, 不再有别的约数, 这个数叫做质数(也叫做素数)。

一个数除了 1 和它本身, 还有别的约数, 这个数叫做合数。

要特别记住: 1 不是质数, 也不是合数。

2. 质因数与分解质因数

如果一个质数是某个数的约数, 那么就称这个质数是这个数的质因数。

把一个合数用质因数相乘的形式表示出来, 叫做分解质因数。

例 1: 把 30 分解质因数。

解答: $30 = 2 \times 3 \times 5$

其中 2、3、5 叫做 30 的质因。

又如 $12 = 2 \times 2 \times 3 = 2^2 \times 3$, 2、3 都叫做 12 的质因数。

例 2: 三个连续自然数的乘积是 210, 求这三个数。

解答: 因为 $210 = 2 \times 3 \times 5 \times 7$

所以可知这三个数是 5、6 和 7。

例 3: 求 240 的约数的个数。

- (A) 6
- (B) 10
- (C) 20
- (D) 24
- (E) 28

解答: 因为: $240 = 2^4 \times 3^1 \times 5^1$

所以 240 的约数的个数是:

$$(4+1) \times (1+1) \times (1+1) = 20$$

所以 240 有 20 个约数。

提示: 一个合数的约数个数, 等于它的质因数分解式中每个质因数的个数(即指数)加 1 的连乘的积。

即如下公式:

$$n = a^x \times b^y \times c^z (a, b, c \text{ 为质数})$$

因子数 $= (x+1)(y+1)(z+1)$

例 4: In a certain game, a large container is filled with red, yellow, green and blue beads worth, respectively, 7, 5, 3, and 2 points each. A number of beads are then removed from the container. If the product of the point values of the removed beads is 147,000, how many red beads were removed?

- (A) 5
(B) 4
(C) 3
(D) 2
(E) 0

翻译: 假如在某一比赛中, 一个大容器中有红色、黄色、绿色和蓝色珠子分别值 7, 5, 3 和 2 点。一些珠子从容器中取出, 假如多出的珠子的点的乘积为 147000, 有多少红色的珠子被移出?

解答: 本题意为取出的珠子点数的乘积为 147000, 则将 147000 拆写成质因子连乘积的形式为:

$$147000 = 2^3 \times 3^1 \times 5^3 \times 7^2$$

由 $r = 7, y = 5, g = 3, b = 2$, 则红色珠子共两个被取出

所以(D) is correct.

提示: 注意抓住问题的核心, 本题和容器及珠子都没有太大关系。

例 5: What is the number of positive factors of 252?

- (A) 9
(B) 15
(C) 18
(D) 21
(E) 27

翻译: 252 因子的个数是多少?

解答: 要想正确得出一个数的不同正因子的数目, 先把该数写成质数因子的连乘积的形式, 如题 $252 = 2^2 \times 7 \times 3^2$

因子数是指指数加 1 的和相乘, 所以因子数为 $(2+1)(1+1)(2+1) = 18$ 。

所以(B) is correct.

提示: 本题也可以采用排除法, 自然数 (除完全平方数, 完全四次方数外), 因子数都是偶数, 因此 A、B、D、E 都可以排除。

例 6: A positive integer with exactly two different divisors greater than 1 must be

- (A) a prime
(B) an even integer
(C) a multiple of 3
(D) the square of a prime
(E) the square of an odd integer

翻译: 仅有两个不同的大于 1 的因子的正整数一定是:

解答: 本题的问题须仔细理解, 意为一个正整数只有两个不同的大于 1 的因子, 这个正整数一定是什么样的数。可以使用排除法:

- (A) 质数只有 1 个大于 1 的因子, 而不是 2 个。
(B) 找出 9 有 3 和 9 两个不同的大于 1 的因子, 而 9 不是偶数。
(C) 可以找出例如 81 这样的 3 的倍数, 其大于 1 的因子有 4 个。
(D) 找不出任何反例。具有两个不同大于 1 的因子的数, 共有 3 个因子, 而因子数求解公式中只有质数的平方才具有 3 个因子。
(E) 可找出 9 是一个奇数, 其平方 81 的大于 1 的不同因子有 4 个。

所以(D) is correct.

3.1.7 奇数和偶数

例 1: $1+2+3+\dots+1993$ 的和是奇数? 还是偶数?

解答: 此题可以利用高斯求和公式直接求出和, 再判别和是奇数, 还是偶数。但是如果从加数的奇、偶个数考虑, 利用奇偶数的性质, 同样可以判断和的奇偶性。此题可以有两种解法。

方法 1: 因为 $1+2+3+\dots+1993$

$$= \frac{(1+1993) \times 1993}{2} = 997 \times 1993$$

又因为 997 和 1993 是奇数, 奇数 \times 奇数=奇数, 所以原式的和是奇数。

方法 2: 因为 $1993 \div 2 = 996 \dots 1$, 所以 1~1993 的自然数中, 有 996 个偶数, 有 997 个奇数。因为 996 个偶数之和一定是偶数, 又因为奇数个奇数之和是奇数, 所以 997 个奇数之和是奇数。因为偶数+奇数=奇数, 所以原式之和一定是奇数。

例 2: If x and y are different prime numbers, each greater than 2, which of the following must be true?

- I. $x+y \neq 91$
- II. $x-y$ is an even integer.
- III. $\frac{x}{y}$ is not an integer.

- (A) II only
- (B) I and II only
- (C) I and III only
- (D) II and III only
- (E) I, II, and III

翻译: 假如 x 和 y 是不同的质数, 且每一个都大于 2, 下列哪一个一定正确?

解答: I. $x+y \neq 91$ 必然成立, 因为 91 必须由一个偶数和一个奇数得到, x, y 都不可能为偶数。

II. $x-y$ 是偶数必然成立, 因为 x, y 都一定是奇数。

III. $\frac{x}{y}$ 不是整数也必然成立, 因为 x, y 均为质数且奇数。

所以(E) is correct.

提示: 本题综合了奇数和偶数的性质及质数和合数的性质, 值得考生注意。

例 3: If $n = p + r$, where n, p , and r are positive integers and n is odd, does p equal 2?

- (1) p and r are prime numbers.
- (2) $r \neq 2$

翻译: 假如 $n = p + r$, 且 n, p, r 均为正整数, n 是奇数, $p=2$ 吗?

- (1) p 和 r 是质数。
- (2) $r \neq 2$

解答: (1)中 p 和 r 是质数, 无法得到 p 是否等于 2; (2)中 $r \neq 2$ 。也无法说明 p 是否等于 2; (1)+(2)能够说明 p 一定等于 2, 因为 $n = p + r$ 且 n 奇数, 所以 p 和 r 中必为一奇一偶, 而 p, r 又都为质数, 则其中一个数必为 2, 由 $r \neq 2$ 则 $p=2$ 。所以(C) is correct.

3.1.8 分数和小数

GMAT 考试要求考生掌握非常熟练的分数和小数间的加减乘除运算。这是我国考生的强项, 在此不再一一赘述。重点介绍一下有关分数和小数的英文术语。

1. 分数

In a fraction $\frac{a}{b}$, ($b \neq 0$), a is the numerator (分子) and b is the denominator (分母). The denominator of a fraction can never be 0, because division by 0 is not

defined.

2. 小数和两种计数法

In the decimal (十进制) system, the position of the period or decimal point determines the place value of the digits. For example, the digits in the number, 8,796.435 have the following place values:

8---thousands' digit	(千位数字)
7---hundreds' digit	(百位数字)
9---tens' digit	(十位数字)
6---ones or units' digit	(个位数字)
4---tenths' digit	(十分位数字)
3---hundredths' digit	(百分位数字)
5---thousandths' digit	(千分位数字)

注意: digit 是“数字”, 即 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 这十个阿拉伯数字; number 是“数”, 如 898 是一个数, 由 3 个 digit 组成。

Scientific Notation(科学计数法):

Decimals are expressed as the product of a number with only one digit to the left of the decimal point and a power of 10.

例如: 0.000486 可用科学计数法表示为: 4.86×10^{-5} 。

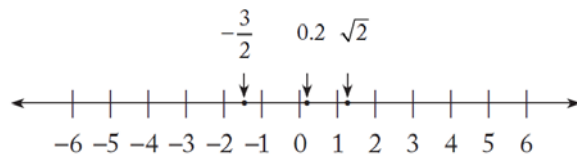
8,245,000 可用科学计数法表示为: 8.245×10^6 。

3.1.9 实数

1. 基本概念

(1) Number Line(数轴)

规定了原点、方向和单位长度的直线。数轴上的点和实数一一对应。从原点出发朝正方向的射线上的点对应正数, 相反方向的射线上的点对应负数, 原点对应数为零。如下图所示:



(2) Absolute Value(绝对值)

某数在数轴上与零点之间的距离称为该数的绝对值, 例如: $|-3| = 3$ 。

(3) Rational Numbers(有理数)

正整数、负整数、正分数、负分数以及零统称为有理数。有理数可以写成 $\frac{m}{n}$ 的形式, 其中 m 和 n 都是整数, 且 n 不等于零。

(4) Irrational Numbers(无理数)

不循环的无限小数。例如用正方形的一边去度量它的对角线时, 所得的比值 $\sqrt{2}$ 就是一个无理数, 写成小数 1.414……时是无限不循环的。又如, 圆周率 $\pi = 3.141592653……$, 也是一个无理数。

(5) Positive and Negative Numbers(正数和负数)

All real numbers except zero are either positive or negative.

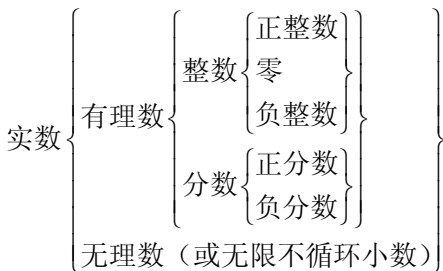
注: 零既不是正数也不是负数。

(6) Real Number(实数)

有理数和无理数统称为实数。与实数相对的为虚数(imaginary numbers)。虚数的内容在 GMAT 考试中不作要求。

All real numbers correspond to points on the number line (数轴) and all points on the number line correspond to real number.

2. 实数的分类



3. 实数的性质

(1) 有理数对四则运算是封闭的(零不能作除数), 而两个无理数的和、差、积和商却不一定是无理数。

(2) 有理数和无理数之间的运算有以下的规律:

有理数 \pm 无理数 = 无理数

非零有理数 \times 无理数 = 无理数

$\frac{\text{非零有理数}}{\text{无理数}} = \text{无理数}$

$\frac{\text{无理数}}{\text{非零有理数}} = \text{无理数}$

(3) 有理数与无理数集无公共元素, 即有理数 \neq 无理数。

(4) 有理数与无理数集具有稠密性(即实数集和数轴上所有组成的集合的一一对应的)和有序性(即可以比较大小)。

(5) 如果 x , y 和 z 都是实数, 则关于 x , y 和 z 有下列性质成立:

① $x+y = y+x$, 和 $xy = yx$

② $(x+y)+z = x+(y+z)$, 和 $(xy)z = x(yz)$

③ $x(y+z) = xy+yz$

④ 如果 x 和 y 都是正数, 那么 $x+y$ 和 xy 都是正数。

⑤ 如果 x 和 y 都是负数, 那么 $x+y$ 是负数, xy 是正数。

⑥ 如果 x 是正数, y 是负数, 那么 xy 是负数。

⑦ 如果 $xy = 0$, 那么 $x = 0$ 或 $y = 0$

⑧ $|x+y| \leq |x|+|y|$

⑨ 在实数中互为相反数的两个数的和为零之, 若两数的和为零, 那么这两数必互为相反数。

3.1.10 比率和比例

比率(Ratio)和比例(Proportion)不会以单独知识点的形式出现考题。但往往会渗透在其他知识点题目, 比如概率和应用题中。这里我们提醒一下大家对比例比率的英文表达方式的读解。如:

How much is the ratio of the number of months with exactly 30 days to the number with exactly 31 days?

答案应该是 $4/7$ 。请牢记 the ratio of A to B 的表达方式。

除此以外, 百分比(Percent)也是一个经常出现的概念。请注意百分比变化(percent change)如百分比升高(percent increase)和百分比下降(percent decrease)的表述方式。

3.1.11 乘方与开方

1. 乘方(Powers)

这种求 n 个相同因数 k 的积的运算, 叫做乘方。在 a^n 中, a 叫做底数(base), n 叫做指数(exponent), a^n 读作 a 的 n 次方(nth power of a)。二次方也叫平方(square), 三次方也叫立方(cube)。

正数的任何次幂都是正数; 负数的奇次幂是负数, 负数的偶次幂是正数。

2. 开方(Roots)

一般地, 如果一个数的平方等于 a , 这个数就叫做 a 的平方根(Square Roots), 换句话说, 如果 $x^2 = a$, 则 x 就叫做 a 的平方根。

一般来说, 一个正数有两个平方根, 这两个平方根互为相反数, 零的平方根为零。在式子 $\pm\sqrt[n]{a}$ 中, a 叫做被开方数, n 叫做根指数。

正数 a 的正的平方根, 也叫做 a 的算术平方根; 零的平方根也叫做零的算术平方根, 因此零的算术平方根仍旧为零。

立方根叫做 **cube roots**, 正数的立方根为正, 负数的立方根为负。

3. 乘方的基本性质

$$(1) a^m a^n = a^{m+n}$$

$$(2) a^m \div a^n = a^{m-n}$$

$$(3) (a^m)^n = a^{mn}$$

$$(4) a^{-m} = \frac{1}{a^m} (m > 0, a \neq 0)$$

$$(5) a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \left(\frac{m}{n} \text{ 为最简分数, 当 } n \text{ 为正偶数时, } a^m \text{ 必为非负数}\right)$$

$$(6) (a \cdot b)^m = a^m \cdot b^m$$

$$(7) \frac{a^x}{b^x} = \left[\frac{a}{b}\right]^x$$

$$(8) a^0 = 1, (a \neq 0)$$

3.2 算术进阶

3.2.1 集合和统计问题

1. 集合

具有某种属性的事物的全体成为集合 (Set), 它一般由一组数或其他符号组成。组成集合的每个事物成为该集合的元素 (Element)。如果 S 是一个有限数量的集合, 那么 $|S|$ 被定义为元素的数目。

例如: $S = \{2, 7, 17, 25\}$, 则 $|S| = 4$ 。

集合之间的关系:

(1) 并集(Union)

集合 A 和 B 的并集是指两个或多个集合中的所有元素, 对两个集合 A 、 B 可表示为: $A \cup B$ 。

(2) 交集(Intersection)

集合 A 和 B 的交集是指两个或多个集合中的所有共同元素, 对两个集合 A 、 B 可表示为: $A \cap B$ 。

(3) Disjoint or Mutually Exclusive

指两个集合中没有共同元素。

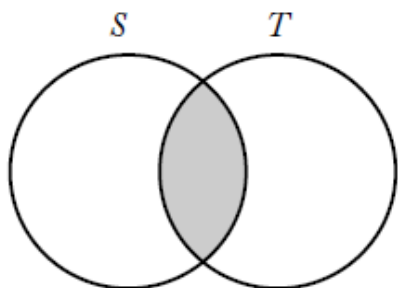
(4) 子集(Subset)

如果一个集合中的所有元素都是另一个集合的组成元素, 则这个集合是另一个集合的子集(subset)。一个集合可以是它自己的子集。

(5) 非集

非某集合元素组成的集合, 称为这集合的非, 对非集合 A 可记作 \bar{A} 。

集合之间的关系往往可以用韦恩图 (Venn Diagram) 来表示。



2. 集合的性质

关于两个集合的容斥原理: 集合 A 与 B 的并的元素个数, 等于集合 A 的元素个数与集合 B 的元素个数的和, 减去集合 A 与 B 的交的元素个数。

3. 统计问题

(1) **Arithmetic mean or Average (算术平均数)**: in a set of a measurements, the sum of the measurements by n .

(2) **Geometric Average (几何平均数)**:

几何平均数为 n 个数的乘积开 n 次方。

例如: a, b, c, d, f 五个数的几何平均数为 $\sqrt[5]{abcdf}$ 。

注: 算术平均数总是大于或等于几何平均数, 对于两个数的情况则有如下式子成立:

$$\frac{a+b}{2} \geq \sqrt{ab}, \text{ 当 } a=b \text{ 时此式的等号成立。}$$

立。

(3) **Middle (中数)**: the middle measurement after the measurements are ordered by size (or the average of the two middle measurements if the number of measurements is odd).

注: 要得到 n 个数的中数, 首先将 n 个数按从大到小的顺序排序, 如果 n 是偶数, 中数被定义为中间的那两个数的平均值。当 n 较大时, 中数被定义为 50% 的线所通过的有序排列中的位置的那个数。也就是说, 如

果 n 的具体数值不确定而为百数, 那么第 50% 的对象所对准的那一个数就是中数。

(4) **Mode (众数)**: the measurement that appears most frequently in a set.

(5) **Range (值域)**: the difference between the greatest measurement and the smallest measurement.

注: Range 是表明数的分布的量, 其被定义为最大值与最小值的差。

(6) **Standard deviation (标准方差)**: a measure of dispersion among members of a set.

具体地说, 有 n 个数分别为 x_1, x_2, \dots, x_n , 这 n 个数的算术平均值为 \bar{x} , 那么这 n 个数的标准方差可用下式表示: Standard

$$\text{deviation} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}.$$

例 1: 求在 1 至 100 的自然数中能被 3 或 7 整除的数的个数。

- (A) 14
(B) 33
(C) 43
(D) 47
(E) 50

解答: 解这类问题时首先要知道在一串连续自然数中能被给定整数整除的数的个数规律是: 在 n 个连续自然数中有且仅有一个数能被 n 整除, 根据这个规律我们可以很容易地求出在 1 至 100 中能被 3 整除的个数为 33 个, 被 7 整除的数的个数为 14 个, 而其中被 3 和 7 都能整除的数有 4 个, 因而得到

设 $A = \{\text{在 } 1 \sim 100 \text{ 的自然数中能被 } 3 \text{ 整除的数}\}$, $B = \{\text{在 } 1 \sim 100 \text{ 的自然数中能被 } 7 \text{ 整除的数}\}$, 则 $A \cap B = \{\text{在 } 1 \sim 100 \text{ 的自然数中能被 } 21 \text{ 整除的数}\}$ 。

因为 $100 \div 3 = 33 \cdots 1$, 所以 $|A| = 33$ 。
 因为 $100 \div 7 = 14 \cdots 2$, 所以 $|B| = 14$ 。
 因为 $100 \div 21 = 4 \cdots 16$, 所以 $|A \cap B| = 4$ 。
 由容斥原理的公式(1):

$$|A \cup B| = 33 + 14 - 4 = 43$$

因此在 1~100 的自然数中能被 3 或 7 整除的数有 43 个。所以(C) is correct。

例 2: 某校有学生 960 人, 其中 510 人订阅“中国少年报”, 330 人订阅“少年文艺”, 120 人订阅“中小学数学教学报”; 其中有 270 人订阅两种报刊, 有 58 人订阅三种报刊, 问这个学校中没有订阅任何报刊的学生有多少人?

- (A) 0
 (B) 120
 (C) 180
 (D) 212
 (E) 328

解答: 设 $A = \{\text{订“中国少年报”的学生}\}$
 $B = \{\text{订“少年文艺”的学生}\}$
 $C = \{\text{订“中小学数学教学报”的学生}\}$
 $I = \{\text{全校学生}\}$
 则问题是要求 $A \cup B \cup C$ 在 I 中的补集 $\overline{A \cup B \cup C}$ 所含元素的个数:
 $|\overline{A \cup B \cup C}| = 960 - |A \cup B \cup C|$
 $= 960 - (410 + 330 + 120 - 270 + 58)$
 $= 212(\text{人})$

全校有 212 名学生没订阅任何报刊。
 所以(D) is correct。

Example 3 and 4 refer to the following information:

In a marketing survey for products A, B, and C, 1,000 people were asked which of the products, if any, they use. The three circular regions in the diagram below represent the numbers of people who use products A, B,

and C, according to the survey results. Of the people surveyed, a total of 400 use A, a total of 400 use B, and a total of 450 use C.

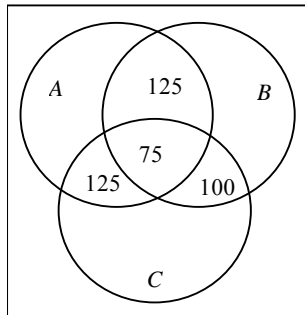


图 3.1

例 3: How many of the people surveyed use exactly one of the products?

- (A) 75
 (B) 100
 (C) 150
 (D) 250
 (E) 325

翻译: 例 3 和例 4 根据下面的信息:

在一个对于产品 A, B, C 的市场营销的调查中, 1000 人被问及他们使用哪一种产品。上图 3.1 中 3 个圆形区域代表使用产品 A, B, C 的人数, 根据调查的结果, 在被调查的人中, 共有 400 人使用 A, 400 人使用 B, 450 人使用 C。

例 3: 调查中有多少人只使用一种产品?

解答: 由上面的集合图, 仅使用一种产品的人一定是不含数字的区域, 单独使用 A 产品有的人:

$$400 - 125 - 125 - 75 = 75$$

单独使用 B 产品的人有:

$$400 - 125 - 75 - 100 = 100$$

单独使用 C 产品的人有:

$$450 - 125 - 75 - 100 = 150$$

所以仅使用一种产品的人有：

$$75 + 100 + 150 = 325$$

所以(E) is correct。

例 4: What percent of the people surveyed use product A or product B or both, but not product C?

(A) 12.5%

(B) 17.5%

(C) 30%

(D) 40%

(E) 60%

翻译：调查中有百分之多少的人或者使用 A，或者使用 B，或者两者同时使用，但不使用 C？

解答：使用 A 但不使用 C 的人有：

$$400 - 125 - 75 = 200$$

使用 B 但不使用 C 的人有：

$$400 - 75 - 100 = 225$$

使用 A 或 B 或两者同时使用的人有：

$$200 + 225 - 125 = 300$$

所以所占百分比为：

$$300/1000 = 30\%$$

(C) is correct。

提示：考生可以借助图形简化复杂集合问题。

例 5: If X and Y are sets of integers, $X \triangle Y$ denotes the set of integers that belong to set X or set Y , but not both. If X consists of 10 integers, Y consists of 18 integers, and 6 of the integers are in both X and Y , then $X \triangle Y$ consists of how many integers?

(A) 6

(B) 16

(C) 22

(D) 30

(E) 174

翻译：假如 X 和 Y 是整数的集合， $X \triangle Y$ 定义为属于集合 X 或属于集合 Y ，但不是两者共同拥有的整数集合。假如 X 中包括 10 个整数， Y 中包括 18 个整数，并且有 6 个整数同时在 X 和 Y ，那么 $X \triangle Y$ 包括多少个整数？

解答： $X \triangle Y$ 中的整数数目为：

$$10 + 18 - 6 - 6 = 16$$

所以(B) is correct。

例 6: In each production lot for a certain toy, 25 percent of the toys are red and 75 percent of the toys are blue. Half the toys are size A and half are size B. If 10 out of a lot of 100 toys are red and size A, how many of the toys are blue and size B?

(A) 15

(B) 25

(C) 30

(D) 35

(E) 40

翻译：在某一玩具的每个产品生产地，25%的玩具是红色，75%的玩具是蓝色。一半的玩具是 A 型的，一半的玩具是 B 型的。假如一批 100 个玩具中的 10 个是红色并且是 A 型，多少玩具是蓝色并且是 B 型？

解答：由 100 个玩具中 10 个是红色并且是 A 型，则有 15 是红色 B 型，而 B 型共 50 个，所以蓝色 B 型有 35 个。

所以(D) is correct。

例 7: For the positive numbers n , $n + 1$, $n + 2$, $n + 4$ and $n + 8$, the mean is how much greater than the median?

(A) 0

(B) 1

(C) $n + 1$ (D) $n + 2$ (E) $n + 3$

翻译: 对于正整数 $n, n + 1, n + 2, n + 4$ 和 $n + 8$, 则算术平均值比中数大多少?

解答: 对于这 5 个数的算术平均值为:

$$\frac{n + (n + 1) + (n + 2) + (n + 4) + (n + 8)}{5}$$

$$= n + 3$$

而 5 个数的中数为 $n + 2$, 则算术平均数比中数大 1, 所以(B) is correct.

提示: 本题考查的是统计问题中的两个概念——算术平均值和中数。

例 8: The geometric mean of 4 and 16 is how much less than the average (arithmetic mean) of 4 and 16?

(A) 0 (B) 1

(C) 2 (D) 3

(E) 4

翻译: 4 和 8 的几何平均值比其算术平均值小多少?

解答: 4 和 8 的几何平均值为 $\sqrt{4 \times 16} = 8$

$$4 \text{ 和 } 16 \text{ 算术平均值为 } \frac{4 + 16}{2} = 10$$

所以(C) is correct.

提示: 有这么一个规律: 算术平均值总是大于等于几何平均值, 等号仅仅在两数相等时才成立。

例 9: The incomplete table below shows a distribution of scores for a class of 20 students. If the average (arithmetic mean) score for the class is 78, what is the median score of 20 students?

Score	Number of Students
83	5
70	6
92	3
	5
64	1

图 3.2

(A) 73

(B) 75

(C) 77

(D) 79

(E) 81

翻译: 未完成的图 3.2 展示了一个班级 20 名学生的分数分配。假如该班级的平均分是 78 分, 这 20 名学生分数的中数是多少?

解答: 由于有一个分数未知所以需首先求出未知分数:

设未知分数为 x , 则得到如下方程

$$78 = (x \times 5 + 83 \times 5 + 70 \times 6 + 92 \times 3 + 64) / 20$$

$$\text{所以 } x = 77$$

如上图 3.2 所示有 3 个 92 分, 5 个 83 分, 5 个 77 分, 中数是第 10 个和第 11 个数的算术平均值, 而第 10 人和第 11 人的得分都是 77 分, 则中数也是 77 分, 所以(C) is correct.

3.2.2 排列组合

$$C_n^1 = P_n^1 = n(n \geq 1)$$

1. 阶乘

n 个自然数 $1, 2, 3, \dots, n$ 的乘积称为 n 的“阶乘”，记作 $n!$ 。例如： $4! = 4 \times 3 \times 2 \times 1$ 。零的阶乘规定为 1 ，即 $0! = 1! = 1$ 。

2. 排列

排列分为两种，非重复的排列问题和可重复的排列问题。前者简称排列问题。这个问题的一般提法是：从 n 个不同的元素 $a_1, a_2, a_3, \dots, a_n$ 中，无放回地任取 $m(1 \leq m \leq n)$ 个按照一定的顺序排成一列，问这样的排列共有多少种？这样的排列总数记为 P_n^m 。

3. 可重复的排列

这个问题的一般提法是：从 n 个不相同的元素 $a_1, a_2, a_3, \dots, a_n$ 中，有放回地任取 m 次，每次取一个，所得到不同的序列共有多少种？这种排列共有 n^m 种。

4. 组合

从 n 个不同元素中，任取 m 个元素并成一组，叫做从 n 个不同的元素中取出 m 个元素的一个组合，用符号 C_n^m 表示。

5. 排列与组合的公式

$$\textcircled{1} P_n^m = \frac{n!}{(n-m)!} (1 \leq m \leq n)$$

$$\textcircled{2} C_n^m = \frac{n!}{m!(n-m)!} (1 \leq m \leq n)$$

$$\textcircled{3} C_n^m = C_n^{n-m} (1 \leq m \leq n)$$

$$\textcircled{4} C_{n+1}^m = C_n^m + C_n^{m-1}$$

$$\textcircled{5} 0! = 1, C_n^0 = P_n^0 = 1,$$

6. 加法原则和乘法原则

加法原则：做一件事，完成它可以有 n 类办法，在第一类办法中有 m_1 种不同的方法，在第二类办法中有 m_2 种不同的方法……，在第 n 类办法中有 m_n 种不同的方法，那么完成这件事共有 $N = m_1 + m_2 + \dots + m_n$ 种不同的方法(在表达中用“或，or”时即为加法原则)。

例 1：某天从 A 地到 B 地，可乘汽车，也可乘火车，还可乘飞机，一天中，汽车有 5 班，火车有 4 班，飞机有 2 班，问一天中 A 地到 B 地共有多少种走法？

解答：根据加法原则可知共有 $5+4+2=11$ 种走法。

乘法原则：做一件事，完成它需

分为 n 个步骤，做第一步有 m_1 种不同的方法，做第二步有 m_2 种不同的方法……，做第 n 步有 m_n 种不同的方法，则完成这件事共有 $N = m_1 \times m_2 \times \dots \times m_n$ 种不同的方法。

例 2：由 A 到 B 有 3 条路，由 B 到 C 有 4 条路，问由 A 经 B 到 C 有多少种不同的走法？

解答：根据乘法原则可知共有 $3 \times 4 = 12$ 种不同的走法。

例 3：由数字 1、2、3、4、5、6 共可组成多少个没有重复数字的四位奇数？

(A) 60

(B) 64

(C) 120

(D) 180

(E) 360

解答: 要组成四位数, 需一位一位地确定各个数位上的数字, 即分四步完成, 由于要求组成的数是奇数, 故个位上只有能取 1、3、5 中的一个, 有 3 种不同的取法; 十位上, 可以从余下的五个数字中取一个, 有 5 种取法; 百位上有 4 种取法; 千位上有 3 种取法, 故可由乘法原理解决。由 1、2、3、4、5、6 共可组成 $3 \times 4 \times 5 \times 3 = 180$ 个没有重复数字的四位奇数。

所以(D) is correct.

例 4: Pat will walk from intersection X to intersection Y along a route that is confined to the square grid of four streets and three avenues shown in the map above. How many routes from X to Y can Pat take that have the minimum possible length?

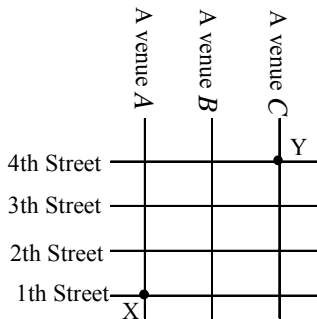


图 3.3

- (A) Six
- (B) Eight
- (C) Ten
- (D) Fourteen
- (E) Sixteen

翻译: P 将从相交点 X 徒步到相交点 Y, 沿着左图所示的 4 条横向街道和 3 条纵向街道所构成的路线, 且街道与街道相交成正方形, 从 X 到 Y 最短可能长度的路线数目是多少?

解答: 从 X 到 Y 的最短长度为 5 个(如图 3.3

所示)正方形的边长, 但路线数目该怎么计算呢? 有些考生通过硬数将答案数出, 这样不但可能发生错误, 当路线进一步复杂时, 这种方法就不适用了, 所以需体会本题正确的计算方法:

要想使路线最短, 经过 2, 3, 4 的顺序一定是固定的, 而经过 B, C 的顺序也肯定固定, 否则不可能路线最短, 且必然经过 2, 3, 4, B, C 这 5 条直线, 所以本题转化为 2, 3, 4, B, C 这 5 个元素的排列, 且满足顺序分别为 2, 3, 4 和 B, C, 则总的排列可能通过两种方法得到:

- ① 5 个位置中选出两个位置给 B, C 且满足 B 在 C 前, 则为 C_5^2 , 剩下 3 个位置也必然按顺序为 2, 3, 4 这种惟一可能, 所以 $C_5^2 = 10$ 。
- ② 5 个位置中选出 3 个位置给 2, 3, 4 且必须满足 2 在 3 之前, 3 在 4 之前, 则为 C_5^3 , 乘下两个位置也必然按顺序 B 在 C 的前面这种惟一可能, 所以 $C_5^3 = 10$ 。

所以(C) is correct.

提示: 本题的关键是将题目转化为适当的排列组合问题。当考生实在不会用排列组合计算时, 硬将线数数出且采用排除法不失为一个好的解决方法。

例 5: How many three-digit numerals begin with a digit that represents a prime number and end with a digit that represents a prime number?

- (A) 16
- (B) 80
- (C) 160
- (D) 180
- (E) 240

翻译:有多少3位数以一个质数开头并且以一个质数结尾?

解答:一位数中为质数的有2, 3, 5, 7四个, 而该3位数的首位和末位都是质数, 而中间的数可为0至9中10个数中的任一个, 所以具有该特征的3位数共有:

$$C_4^1 C_{10}^1 C_4^1 = 160$$

所以(C)is correct.

例 6: Three stacks containing equal numbers of chips are to be made from 9 red chips, 7 blue chips, and 5 green chips. If all of these chips are used and each stack contains at least 1 chip of each color, what is the maximum number of red chips in any one stack?

- (A) 7
- (B) 6
- (C) 5
- (D) 4
- (E) 3

翻译:3堆含有相同数目的筹码由9个红色筹码、7个蓝色筹码和5个绿色筹码组成。假如所有筹码都被使用, 并且每堆至少含有每种颜色中的一个筹码, 任一堆中红色筹码的最大数目是多少?

解答:所有筹码的数目为 $9+7+5=21$, 每堆中有7个筹码, 因为每堆中至少有某颜色的一个筹码, 所以其他两堆中每一堆最少有一个红色筹码, 而这一堆中有至少有一个蓝色一个绿色筹码, 所以红色筹码的最大数目为5个。

所以(C)is correct.

例 7: If 10 persons meet at a reunion and each person shakes hands exactly once with each of the others, what is the total number of handshakes?

- (A) $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
- (B) 10×10
- (C) 10×9
- (D) 45
- (E) 36

翻译:假如10个人在一次聚会中相遇, 每人与另一人握一次手, 握手的总数是多少?

解答:两个人握手, 则为从10人中任取两人, $C_{10}^2 = 45$

所以(D)is correct.

例 8: Each person at a party shook hands exactly once with each of the other people at the party. There was a total of 21 handshakes exchanged at the party. What is the number of people at the party?

- (A) 5
- (B) 6
- (C) 7
- (D) 8
- (E) 9

翻译:一次舞会中的每个人恰与参加舞会的其他人中的每一位握一次手, 在舞会中共握了21次手。

解答:设舞会上有 n 个有, 则握手次数为

$$C_n^2 = \frac{n(n-1)}{2} = 21$$

$n = 7$ 所以(C)is correct.

3.2.3 概率

1. 概率 (Probability 或者 Possibility)

亦称“或然率”、“几率”, 某一类事件在相同的条件下可能发生也可能不发生, 这类事件称为“随机事件”(random occurrence)。概率就是用来表示随机事件发生的可能性大小的一个量。很自然地把必然

发生的事件的概率定为 1, 并把不可能发生的事件的概率定为 0, 而一般随机事件的概率是介于 0 和 1 之间的一个数。

任何随机事件 A 在相同的条件下发生的概率介于 0 和 1 之间, 即 $0 \leq P(A) \leq 1$; 必须事件 U 和不可能事件 V 发生的概率分别为 1 和 0, 即 $P(U) = 1, P(V) = 0$ 。

2. 概率的性质

(1) 等可能性事件的概率: 如果一次试验中共有 n 种等可能出现的结果, 其中事件 A 包含的结果有 m 种, 那么事件 A 的概率 $P(A) = \frac{m}{n}$ 。

例 1: 有 7 个奇数, 5 个偶数, 从这 12 个数中任取一个是奇数的概率?

解答: 这 12 个数任取 1 个有 12 种可能结果, 取奇数的结果为 7 种, 因此其概率为 $\frac{7}{12}$ 。

(2) 互斥事件发生的概率: 如果事件 A_1, A_2, \dots, A_n 彼此互斥, 那么事件 A_1, A_2, \dots, A_n 中有一个发生的概率为这 n 个事件分别发生的概率的和, 即 $P(A_1, A_2, \dots, A_n) = P(A_1) + P(A_2) + \dots + P(A_n)$, 也即用“or, 或”表达。(注: 所谓互斥是指任两个之间都不可能同时发生)。

例 2: 在 12 个球中, 8 个是一等品, 3 个是二等品, 1 个是三等品, 求任取一个球是一等品或是二等品的概率?

解答: 这 12 个球中, 取一等品的概率为 $\frac{8}{12}$, 取二等品的概率为 $\frac{3}{12}$, 这两个事件是互斥的, 因若取了一等品就不会是二等品, 取了二等品就不会是一等品, 因此根据上边公式可知本题的答案为 $\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$ 。

(3) 相互独立事件同时发生的概率: 如果事件相互独立, 那么 n 个事件同时发生的概率等于每个事件发生的概率的积, 即 $P(A_1, A_2, \dots, A_n) = P(A_1)P(A_2)\dots, P(A_n)$, 也即用“用”或“and”来表达。

例 3: A 坛中有 7 个白球, 3 个黑球, B 坛中有 4 个白球, 5 个黑球, 问从这两个坛中分别摸出一个都是白球的概率?

解答: 从 A 坛中摸出一个白球的概率为 $\frac{7}{7+3}$, 从 B 坛中摸出一个白球的概率为 $\frac{4}{4+5}$, 这两个事件是相互独立的, 互不影响, 则根据上面所述其概率应为 $\frac{7}{10} \times \frac{4}{9} = \frac{14}{45}$ 。

(4) 独立重复试验发生的概率: 如果在一次试验中某事件发生的概率是 P , 那么在 n 次独立重复试验中这个事件恰好发生 K 次的概率为 $P_n(K) = C_n^k \cdot P^k (1-p)^{n-k}$ 。

例 4: 某气象站天气预报准确率为 80%, 求 5 次预报中有 4 次准确的概率?

解答: 设 P 为预报一次, 结果准确的概率, 预报 5 次, 相当于 5 次独立重复试验, 根据上式则有:

$$P_4(4) = C_5^4 \times 0.8^4 \times (1-0.8)^{5-4} \\ = 5 \times 0.8^4 \times 0.2 = 0.4096。$$

例 5: 36 件产品中有 4 件次品, 从中随机取 3 件, 求恰好取到 1 件次品的概率。

- (A) $\frac{C_4^1 \cdot C_{32}^2}{C_{36}^3}$
 (B) $\frac{C_4^1 \cdot C_{36}^2}{C_{36}^3}$
 (C) $\frac{C_4^1 \cdot C_{32}^2}{C_{32}^3}$

$$(D) \frac{C_4^1 \cdot C_{32}^2}{C_{36}^3}$$

$$(E) \frac{C_4^1 \cdot C_{32}^2}{C_{36}^4}$$

解答: 可能取法的总数为 C_{36}^3 , 恰好取到 1 件次品的取法数为 $C_4^1 \times C_{32}^2$, 故所求概率是 $p = \frac{C_4^1 \cdot C_{32}^2}{C_{36}^3}$, 所以(A) is correct.

例 6: 在例 5 条件下, 求至少取到 1 件次品的概率。

解答:

方法 1: 以 A 表示“3 件中至少有 1 件次品”, 则 A 可以写成下面三个两两互斥的事件之和:

$A_1 =$ “3 件中有 1 件次品 2 件正品”;

$A_2 =$ “3 件中有 2 件次品 1 件正品”;

$A_3 =$ “3 件全是次品”

按例 5 的算法可得

$$P(A_1) = \frac{C_4^1 \cdot C_{32}^2}{C_{36}^3} \approx 0.2778;$$

$$P(A_2) = \frac{C_4^2 \cdot C_{32}^1}{C_{36}^3} \approx 0.0269;$$

$$P(A_3) = \frac{C_4^3}{C_{36}^3} \approx 0.0006。$$

按概率的可加性, 得所求概率为

$$\begin{aligned} P(A) &= P(A_1 + A_2 + A_3) \\ &= P(A_1) + P(A_2) + P(A_3) \\ &\approx 0.3053 \end{aligned}$$

方法 2: 利用关于对立事件的性质 5 可得如下更好的解法。 A 的对立事件 $\bar{A} =$ “3 件全是正品”, 而

$$P(\bar{A}) = \frac{C_{32}^3}{C_{36}^3} \approx 0.6947$$

按性质 5 得所求概率为

$$P(A) = 1 - P(\bar{A}) \approx 0.3053。$$

例 7: 抽签口试, 共有 $\alpha + \beta$ 张考签, 每个考生抽 1 张考签, 抽过的不再放回, 考生王某会答其中 α 张, 他是第 k 个抽签者 ($k \leq \alpha + \beta$), 求王某抽到会答考签的概率。

解答: 考虑把 $\alpha + \beta$ 张考签依次抽出来, 抽法总数为 $(\alpha + \beta)!$ 。“王某抽到会答考签”即第 k 张必须且只须是王某会答的 α 张之一, 第 k 张有 α 种抽法, 其他 $\alpha + \beta - 1$ 张有 $(\alpha + \beta - 1)!$ 种抽法; 因此, 组成随机事件“王某抽到会答考签”的抽法数为 $\alpha \times (\alpha + \beta - 1)!$, 所求概率

$$p = \frac{\alpha(\alpha + \beta - 1)!}{(\alpha + \beta)!} = \frac{\alpha}{\alpha + \beta}$$

提示: 值得注意的是, 这概率与 k 无关, 说明考生不管先抽后抽, 抽到答考签的概率都一样。

例 8: In a certain animal population, for each of the first 3 months of life, the probability that an animal will die during that month is $1/10$. For a group of 200 newborn members of the population, approximately how many would be expected to survive the first 3 months of life?

(A) 140

(B) 146

(C) 152

(D) 162

(E) 170

翻译: 在某种动物中, 出生后前 3 个月中的每个月, 动物在那个月死亡的概率是 $1/10$, 对于一组 200 个新成员中, 预计有多少可以在前 3 个月存活下来?

解答: 由死亡的概率为 0.1, 则第一个月后

生存下来 $200(1-0.1)$ 个, 同理第二个月为 $200(1-0.1)^2$, 第三个月为 $200(1-0.1)^3 = 146$ 个。

所以(B) is correct。

例 9: Xavier, Yvonne, and Zelda each try independently to solve a problem. If their individual probabilities for success are $1/4$, $1/2$, and $5/8$, respectively, what is the probability that Xavier and Yvonne, but not Zelda, will solve the problem?

- (A) $\frac{11}{8}$
 (B) $\frac{7}{8}$
 (C) $\frac{9}{64}$
 (D) $\frac{5}{64}$
 (E) $\frac{3}{64}$

翻译: 泽维尔、依冯和赛尔达每个人试图独立地解出一个题目。假如他们每个人成功的概率分别为 $1/4$, $1/2$ 和 $5/8$, 泽维尔和依冯将解出题目而赛尔达将解不出题目的概率是多少?

解答: 泽维尔解出题目的概率为 $1/4$, 依冯的概率为 $1/2$, 而赛尔达解不出题目的概率为 $3/8$, 则泽维尔和依冯解出题目而赛尔达解不出题的概率为:

$$1/4 \cdot 1/2 \cdot 3/8 = 3/64。$$

所以(E) is correct。

例 10: A jar contains only x black balls and y white balls. One ball is drawn randomly from the jar and is not replaced. A second ball is then drawn randomly from the jar. What is the probability that the first ball drawn is black and the second ball drawn is white?

- (A) $[x/(x+y)][x/(x+y)]$
 (B) $[x/(x+y)][(x-1)/(x+y-1)]$
 (C) $xy/(x+y)$
 (D) $[(x-1)/(x+y)][(y-1)/(x+y)]$
 (E) $[x/(x+y)][y/(x+y-1)]$

翻译: 一个罐子中仅含 x 个黑球和 y 个白球, 一个球被随机地从罐子中取出并且不再更换, 然后第二个球从罐子中随机取出。取出的第一个球为黑色且第二个球为白色的概率是多少?

解答: 第一个球为黑球的概率为 $x/(x+y)$, 第二个球为白球的概率为 $y/(x-1+y)$,

所以第一个球为黑色第二个球为白色的概率为:

$$[x/(x+y)][y/(x+y-1)]$$

所以(E) is correct。

例 11: An experiment has three possible outcomes, I , J and K . The probabilities of the outcomes are 0.25, 0.35, and 0.40, respectively. If the experiment is to be performed twice and the successive outcomes are independent, what is the probability that K will not be an outcome either time?

- (A) 0.36
 (B) 0.40
 (C) 0.60
 (D) 0.64
 (E) 0.80

翻译: 一个实验有 3 个可能的结果 I 、 J 和 K 。这些结果出现的概率分别为 0.25、0.35 和 0.4, 假如实验被连续进行 2 次, 并且连续的结果之间相互独立, K 不是

任一次结果的概率有多大?

解答: K 不是第一次的结果概率为:

$$1 - 0.4 = 0.6$$

同理 K 不是第二次的结果的概率也为 0.6, 两次结果之间也相互独立,

所以 $0.6 \times 0.6 = 0.36$ 。

(A) is correct.

3.3 代数

3.3.1 数列

1. 等差数列

如果数列(sequence)从第二项开始, 每一项与前一项的差为常数 d , 则称该数列为“等差数列”(arithmetic sequence), d 称为“公差”(common difference)。等差数列可写成 $a, a+d, a+2d, \dots, a+(n-1)d, \dots$ 的形式。等差数列具有以下性质:

如果 $a_1, a_2, a_3, \dots, a_n, \dots$ 是一个以 a_1 为第一项, d 为公差和 a_n 为第 n 项的等差数列, 则下式成立:

$$\text{I. } a_n = a_1 + (n-1)d$$

II. 前 n 项之和

$$S_n = \frac{n(a_1 + a_n)}{2} = na_1 + \frac{n(n-1)d}{2}。$$

III. M (中项或中数)

(1) 当 n 为偶数时, M 为中间两项的

$$\text{算术平均值 } M = \frac{\frac{a_n + a_{\frac{n}{2}+1}}{2}}{2}。$$

(2) 当 n 为奇数时, M 为中间项

$$M = a_{\frac{n+1}{2}}。$$

2. 等比数列

如果数列从第二项开始, 每一项与前一项的比为常数 q , 则称该数列为“等比数列”(geometric sequence), q 称为“公比”(common ratio)。等比数列可以写成 $a, aq, aq^2, \dots, aq^n, \dots$ 的形式。等比数列具有如下性质:

如果 $a_1, a_2, a_3, \dots, a_n, \dots$ 是一个以 a_1 为第一项, q 为公比和 a_n 为第 n 项的等比数列, 则下式成立:

$$\text{I. } a_n = a_1 q^{n-1}$$

II. 前 n 项的和

$$S_n = \frac{a_1(1-q^n)}{1-q} (q \neq 1), \text{ 或}$$

$$S_n = \frac{a_1 - a_n q}{1 - q} (q \neq 1)$$

III. M(中项或中数)

(1) 当 n 为偶数时, M 为中间两项的几何平均数: $M = \sqrt{a_{\frac{n}{2}} \times a_{\frac{n}{2}+1}}$

(2) 当 n 为奇数时, M 为中间项:

$$M = a_{\frac{n+1}{2}}$$

例 1: 把 210 拆成 7 个自然数的和, 使这 7 个数从小到大排成一行后, 相邻两个数的差都是 5, 那么, 第 1 个数与第 6 个数分别是多少?

解答: 由题可知: 由 210 拆成的 7 个数必构成等差数列, 则中间一个数为 $210 \div 7 = 30$, 所以, 7 个数分别是 15、20、25、30、35、40、45, 即第 1 个数是 15, 第 6 个数是 40。

例 2: $x+y+z=1993$ 有多少组正整数解。

解答: $x=1991$, 则 $y+z=2$, 所以 $y=z=1$ 1 组

$$x=1990, \text{ 则 } y+z=3,$$

$$\text{所以 } \begin{cases} y=1 \\ z=2 \end{cases} \text{ 或 } \begin{cases} y=2 \\ z=1 \end{cases} \quad 2 \text{ 组}$$

$$x=1989, \text{ 则 } y+z=4,$$

$$\begin{cases} y=1 \\ z=3 \end{cases} \begin{cases} y=2 \\ z=2 \end{cases} \begin{cases} y=3 \\ z=1 \end{cases} \quad 3 \text{ 组}$$

$$x=1988, \text{ 则 } y+z=5,$$

$$\begin{cases} y=1 \\ z=4 \end{cases} \begin{cases} y=2 \\ z=3 \end{cases} \quad 4 \text{ 组}$$

$$\dots \begin{cases} y=3 \\ z=2 \end{cases} \begin{cases} y=4 \\ z=1 \end{cases} \quad 4 \text{ 组}$$

$$x=2, \text{ 则 } y+z=1991 \dots \quad 1990 \text{ 组}$$

$$x=1, \text{ 则 } y+z=1992 \dots \quad 1991 \text{ 组}$$

显然, x 不能等于 1992、1993。

所以, 原方程的不同的整数解的组数是: $1+2+3+\dots+1991=1983036$ 。

提示: 本题中运用了分类的思想, 先按照 x 的值分类, 在每一类中, 又从 y 的角度

来分类, 如: $x=1987$ 时, 因为 $y+z=6$, 且 y, z 均为正整数, 所以 y 最小取 1, 最大取 5, 即按 $y=1, 2, 3, 4, 5$ 分类, 每一类对应一组解。因此, $x=1987$ 时, 共 5 组解。

例 3: In a list of numbers, the first number is 3, the second is 4, and each subsequent number is the sum of all the preceding numbers in the list. If the 20th number in the list is x , what is the 25th number in the list?

- (A) $7x$
 (B) $16x$
 (C) $32x$
 (D) $35x$
 (E) $161x$

翻译: 在一个数列中, 首数为 3, 第二项数为 4, 并且下一个数是所有前面数的和。假如第 20 项数是 x , 第 25 项数是多少?

解答: 第 20 项为 x , 则前面 19 项的和也是 x , 第 21 项为 $2x$, 22 项为 $4x$, 23 项为 $8x$, 24 项为 $16x$, 25 项为 $32x$ 。

所以(C) is correct.

错误分析: 如果逐项去求, 那就大错而特错了。数列问题关键是把握项与项之间的关系。

3.3.2 因式分解

因式分解(factoring)在解方程、不等式以及在代数式的运算中具有举足轻重的地位, 灵活运用因式分解的技巧是准确快速解答许多数学题的关键。常见的因式分解方法主要有以下四种:

1. 提取公因式法

如果一个多项式的各项含有公因式,可以把公因式作为多项式的一个因式提出来,用这个因式去除这个多项式,把所得的商作为另一个因式,这种因式分解的方法叫做提取公因式法。此法是分解因式最常用的方法,也是在因式分解时,首先考虑的方法。

提取公因式的基本思维方式是“求同”,为了“求同”,常要对给定的多项式进行适当的恒等变形,创造提取公因式的条件。

2. 运用公式法

在因式分解中,有时需要运用乘法公式(甚至反复应用公式),因式分解常用的公式有:

- (1) $a^2 - b^2 = (a+b)(a-b)$
- (2) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- (3) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- (4) $a^2 + 2ab + b^2 = (a+b)^2$
- (5) $a^2 - 2ab + b^2 = (a-b)^2$
- (6) $a^3 + 3a^2b + 3ab^2 + b^3 = (a+b)^3$
- (7) $a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3$
- (8) $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2$

3. 分组分解法

把多项式的项通过适当分组来分解因式的方法,叫做分组分解法。

运用分解法分解因式时,对多项式恰当分组的要求是:分组后各组能分解因式,并且在各组分解因式的基础上,能完成对整个多项式的因式分解,分组是为进行因式分解创造条件,是搭桥,所以地考虑如何适当分组时,通常要进行尝试和估算,分组的基本方向是“求同”,也即把各项联系起来。

3.3.3 方程

1. 一元二次方程

一元二次方程的通式是 $ax^2 + bx + c = 0 (a \neq 0)$ 。

一元二次方程常见的解法包括因式分解法和求根法。

(1) 因式分解法

利用我们刚刚学过的因式分解的知识,若一元二次方程能够变为以下形式:

$$a(x-x_1)(x-x_2) = 0$$

则方程的解为 $x = x_1$ 或 $x = x_2$

(2) 公式法

一元二次方程 $ax^2 + bx + c = 0 (a \neq 0)$ 的求根公式是

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (b^2 - 4ac \geq 0)$$

$$\Delta = b^2 - 4ac$$

在 $\Delta > 0$ 时有两个不相等的实数根;

在 $\Delta = 0$ 时有两个相等的实数根;

在 $\Delta < 0$ 时没有实数根。

如果 $ax^2 + bx + c = 0 (a \neq 0)$ 的两个根是 x_1, x_2 , 那么:

$$x_1 + x_2 = -\frac{b}{a}, \quad x_1 \cdot x_2 = \frac{c}{a}$$

如果方程为 $x^2 + px + q = 0$, 则:

$$x_1 + x_2 = -p, \quad x_1 \cdot x_2 = q$$

例 1: 求解方程: $2x^2 - 13x + 15 = 0$

解答: 方程可以变形为

$$(2x-3)(x-5) = 0,$$

故方程解为: $x = \frac{3}{2}$ 或 $x = 5$ 。

例 2: 设 x_1, x_2 是方程 $2x^2 - 13x + 15 = 0$ 的两个根, 求 $\frac{x_2}{x_1} + \frac{x_1}{x_2}$ 的值。

解答: 方法 1: 求解方程可得:

$$x_1 = \frac{3}{2}, \quad x_2 = 5$$

$$\text{所以 } \frac{x_2}{x_1} + \frac{x_1}{x_2} = \frac{109}{30}$$

方法 2:

$$\frac{x_2}{x_1} + \frac{x_1}{x_2} = \frac{x_1^2 + x_2^2}{x_1 x_2} = \frac{(x_1 + x_2)^2}{x_1 x_2} - 2$$

根据根与系数的关系:

$$x_1 + x_2 = \frac{13}{2}$$

$$x_1 x_2 = \frac{15}{2}$$

$$\text{所以 } \frac{x_2}{x_1} + \frac{x_1}{x_2} = \frac{\left(\frac{13}{2}\right)^2}{\frac{15}{2}} - 2 = \frac{109}{30}$$

2. 二元一次方程组

对二元一次方程组解是一般采用消元法, 如下述

$$\begin{cases} a_1 x + b_1 y = c_1 & (1) \\ a_2 x + b_2 y = c_2 & (2) \end{cases}$$

方程①变为 $x = \frac{c_1 - b_1 y}{a_1}$ 将 x 代入②式

中, 从而求得 y , 再将 y 代入①或②式中求得 x , 但需注意以下几点:

(1) 若 $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ 则此时①, ②方程

为等价方程, 方程组有无数解。

(2) 若 $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ 则此时①, ②方程

为矛盾方程, 方程组无解。

(3) 若 $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ 方程组有惟一解。

从上可知, 不可认为只要是方程组就可解得 x, y 的值, 必须对系数加以判断方可断定。

例: 用浓度为 5% 和 53% 的两种烧碱溶液混合配制成浓度为 25% 的烧碱溶液 300 千克, 需用这两种烧碱溶液各多少千

克?

解答: 根据下面的相等关系, 可以列出一个二元一次方程组:

混合前烧碱溶液的重量 = 混合后浓液中所含纯烧碱的重量。

设需用 5% 的烧碱溶液 x 千克, 53% 的烧碱溶液 y 千克, 根据题意, 得

$$\begin{cases} x + y = 300 & (1) \\ 5\%x + 53\%y = 25\% \cdot 300 & (2) \end{cases}$$

化简(2), 得

$$5x + 53y = 7500 \quad (3)$$

(3) $-(1) \times 5$, 得

$$48y = 6000,$$

所以 $y = 125$

把 $y=125$ 代入(1), 得

$$x + 125 = 300,$$

所以 $x = 175$

$$\text{所以 } \begin{cases} x = 175 \\ y = 125 \end{cases}$$

因此需用浓度为 5% 的烧碱溶液 175 千克, 53% 的烧碱溶液 125 千克。

3.3.4 不等式

1. 一元一次不等式

一元一次不等式总可以通过去括号, 去分母、移项, 合并同类项 (combining like term) 化为形式为 $ax > b$ 的不等式, 具解的情况是:

当 $a > 0$ 时, $x > \frac{b}{a}$; 当 $a < 0$ 时, $x < \frac{b}{a}$;

当 $a = 0$ 时, 若 $b < 0$, 则解为一切实数; 若 $b \geq 0$ 时则无解。

例 1: 解关于 x 的不等式 $1 - \frac{2x}{a^2} > \frac{x}{a} + \frac{4}{a^2}$ 。

解答: 这里显然 $a \neq 0$, 由原不等式得:

$$(a+2)x < (a+2)(a-2)$$

当 $a+2 > 0$, 即 $a > -2$ 时, 解集为 $x < a-2$;

当 $a+2 < 0$, 即 $a < -2$ 时, 解集为 $x > a-2$;

当 $a+2 = 0$, 即 $a = -2$ 时, 无解。

2. 一元二次不等式

一元二次不等式的一般形式是

$$ax^2 + bx + c > 0 \text{ 或 } ax^2 + bx + c < 0 (a > 0).$$

一元二次不等式的解法有: 配方法、分析法和图象法。

例 2: 解不等式 $x^2 - 4x + 3 > 0$

解答: **方法 1:** (配方法) 将不等式的左端配方, 得:

$$(x-2)^2 - 1 > 0, \text{ 即 } (x-2)^2 > 1, \text{ 两边同时开方, 得:}$$

$|x-2| > 1$, 所以 $x-2 > 1$ 或 $x-2 < -1$

所以原不等式的解集为 $x > 3$ 或 $x < 1$

方法 2: (分析法) 将不等式左端因式分解,

得: $(x-1)(x-3) > 0$, 通过对因式的符号分析, 将原不等式化为两等价的不等式组。

$$(1) \begin{cases} x-1 > 0 \\ x-3 > 0 \end{cases} \text{ 或 } (2) \begin{cases} x-1 < 0 \\ x-3 < 0 \end{cases}$$

由(1)得 $x > 3$, 由(2)得 $x < 1$

因此原不等式的解集是: $x > 3$ 或 $x < 1$

解法(3): (图象法)

$$\text{令 } x^2 - 4x + 3 = 0,$$

$$\text{所以 } \Delta = 16 - 12 = 4 > 0$$

所以方程有两个不同的根, 解方程, 得

$$x_1 = 1, x_2 = 3.$$

这就是函数 $x^2 - 4x + 3$ 与 x 轴的交点的横坐标。

又因为 $a = 1 > 0$, 开口向上, 作出草图,

由图 3.4 可知 $x^2 - 4x + 3 > 0$ 的解集为 $x < 1$ 或 $x > 3$ 。

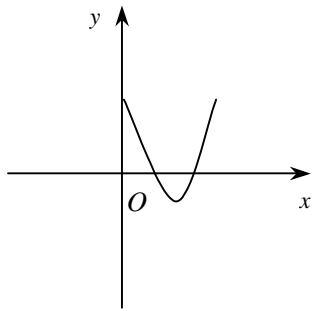


图 3.4

3.3.5 函数

1. 定义

设在某变化过程中的两个变量 (variable) x 和 y , y 随 x 而变化, 而且依赖于 x 。如果变量 x 取某个特定的值, y 依确定的关系取相应的值, 则称 y 是 x 的函数。记作 $y = f(x)$, 其中 x 称为自变量 (independent variable), y 对称为因变量 (dependent variable)。 x 的变化范围称为函数的“定义域”, 与 x 对应的 y 的取值称为“函数值”, 其全体称为函数的“值域”。GMAT 数学考试中会出现诸如括号、圆圈和方框等各种形式的函数表达式。

2. 函数的表达方法

“ $f(x)$ ”和“ $g(x)$ ”仅是函数表达的符号, 函数还可以有其他方式来表达。考题中经常给出一些特殊符号来定义各种函数, 例如: $x \square y = x^2 - y^2$, 那么 \square 表达了 x 与 y 之间的运算关系, $3 \square 2 = 3^2 - 2^2 = 9 - 4 = 5$; 再如 $x^* = \frac{1}{x}$, 则 $\left(\left(\left(\frac{1}{2}\right)^*\right)^*\right)^* = 2$ 。

3. 函数的定义域

函数的定义域是指函数中自变量所允许的取值范围。例如函数 $f(x) = x^3 + 3x^2 - 4$ 的

定义域是全体实数；函数 $g(z) = \frac{\sqrt{5z-2}}{3x+8}$ 的

定义域是 $x \neq -\frac{8}{3}$ ；函数 $f(x) = \sqrt{1-x^2}$ 的定

义域是 $-1 \leq x \leq 1$ ，也可表示为 $[-1, 1]$ 。另外我们还可设定函数的定义域，例如我们可以设定函数 $f(x) = x^3 + 3x^3 - 4$ 的定义域是 $-2 \leq x \leq 3$ 。

3.4 几何-已不考，可不看

3.4.1 平面几何

1. 三角形

① **等腰三角形**：有两条边相等的三角形，其相等的两条边称为腰。

(isosceles triangle)

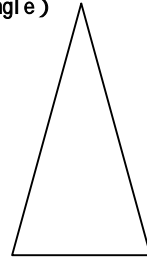


图 3.5

性质：两腰所夹顶点到底边的中点连线必垂直于底边或两腰所夹顶点到底边的垂线必过底边中点。

② **等边三角形**：三条边都相等的三角形。**equilateral triangle**

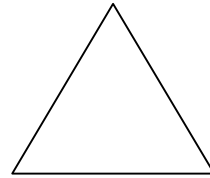


图 3.6

性质：各顶点到底边的中线、高线是重合的，各角都为 60 度。

$$\text{面积} = \frac{\sqrt{3}}{4} a^2 \quad (a \text{ 为边长})$$

$$\text{高} = \frac{\sqrt{3}}{2} a$$

③ **直角三角形**：有一个角为 90 度的三角形。**right triangle**

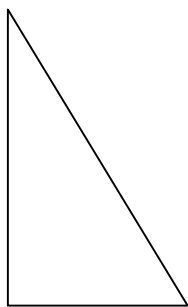


图 3.7

性质: $a^2+b^2=c^2$ (勾股玄定理)

面积 = $\frac{a \times b}{2}$ (a, b 为两直角边)

满足如下三边长的必为直角三角形:

	一	二	三	四	五	六
边一	3	4	5	6	8	10
边二	5	12	13	10	24	26
边三	7	24	25	14	48	50

④ 三角形的性质及推论:

性质:

(A) 三角形中两边之和大于第三边。

(B) 三角形中两边之差小于第三边。

(C) 三角形中大边所对角较大。

(D) 三角形中大角所对边较大。

(E) 三角形一个外角等于与它不相邻的两个内角之和。

推论: (1) 三角形中若最小的两条边的平方和小于第三条边的平方和, 则此三角形必为钝角三角形。

(2) 三角形中若最小的两条边的平方和大于第三条边的平方和, 则此三角形必为锐角三角形。

2. 圆circle

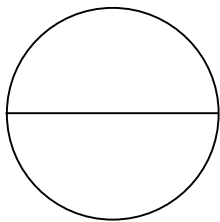


图 3.8

① 有关圆的基本概念:

(1) **Radius(半径):** the distance from a circle's center to any point on the circle.

(2) **Diameter (直径):** the greatest distance from one point to another on the circle.

(3) **Chord(弦):** a line segment connecting two points on the circle.

(4) **Circumference(周长):** the distance around the circle (its "perimeter").

(5) **Arc (弧):** a segment of a circle's circumference (an arc can be defined either as a length or as a degree measure).

(6) **Tangent to a circle (圆的切线):** 一条直线与圆只有一个交点称该直线与圆相切, 交点叫做切点(point of tangency);

(7) **Secant to a circle (圆的割线):** 与圆有两个交点的直线称为圆的割线;

(8) **Central angle (圆心角):** 顶点在圆心上并且两条边是圆的弦的角称为“圆心角”;

(9) **Inscribed angle(圆周角):** 顶点位于圆周上两条边是圆的弦的角称为“圆周角”;

(10) **Sector (扇形):** 圆弧和它对应的圆心角所围成的一部分平面区域称“扇形”。

② 周长和面积求法

周长 = $2 \pi r = \pi R$ (r 为半径, R 为直径)

面积 = $\pi r^2 = \pi R^2/4$

③ 圆的某些性质

A. 在圆中同弧所对的圆心角是圆周角的两倍。

图中 O 为圆心, 点 A, B, C 在圆上, 则 $\angle BOC$ 为圆心角 $\angle BAC$ 为圆周角, 有 $\angle BOC = 2\angle BAC$ 。

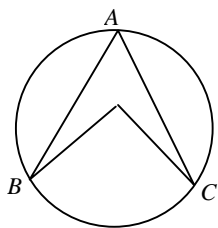


图 3.9

B. 在圆中

$$\frac{\text{弧长}}{\text{圆周长}} = \frac{\text{弧所对应角度}}{360^\circ}$$

例 1: 某圆半径为 2, 求 60 度角所对弧的长度。

解答: 由上则有

$$\frac{\text{弧长}}{2\pi \times 2} = \frac{60}{360} \Rightarrow \text{弧长} = \frac{2}{3}\pi$$

C. 如图 3.10 所示, O 为圆心, AB 连线过点 O , 且 AB 在圆上, 则:

若点 C 在圆外时, $\angle ACB$ 必为锐角。

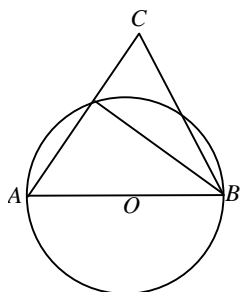


图 3.10

若点 C 在圆上时, $\angle ACB$ 必为直角。

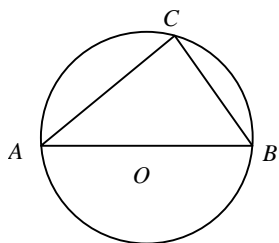


图 3.11

若点 C 在圆内时, $\angle ACB$ 必为钝角。

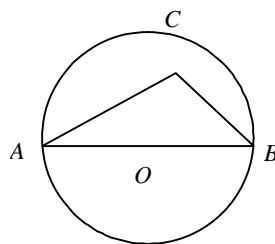


图 3.12

D. 如图 3.13 所示, O 为圆心, 则过圆外一点 A 做圆的切线交圆于点 B , 则 $\angle ABO$ 为直角。

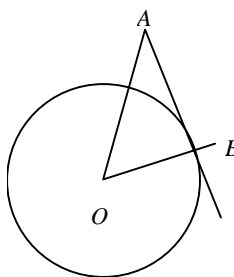


图 3.13

3. 正方形 square

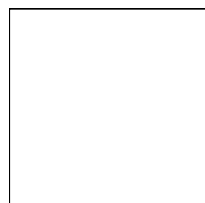


图 3.14

周长 = $4a$

面积 = a^2 (a 为边长)

4. 长方形 rectangle



图 3.15

周长 = $2(a+b)$

面积 = $a \times b$ (a, b 为长和宽)

5. 平行四边形: Parallelogram

对边平行的四边形



图 3.16

6. **菱形**: 四边相等的平行四边形, 菱形的对角线是互相垂直的。diamond

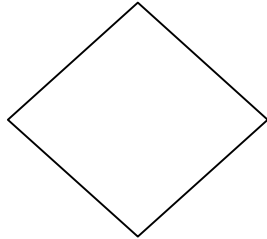


图 3.17

周长 = $4a$ (a 为边长)

面积 = 两条对角线乘积的一半

7. **梯形 trapezium**

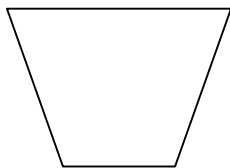


图 3.18

面积 = $\frac{(\text{上底} + \text{下底}) \times \text{高}}{2}$

8. **平行的判定**

A. 内错角相等, 两直线平行

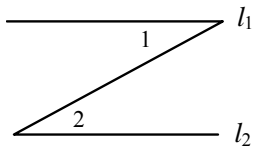


图 3.19

$\angle 1$ 和 $\angle 2$ 为内错角, 若 $\angle 1 = \angle 2$, 则 $l_1 // l_2$

B. 同位角相等, 两直线平行

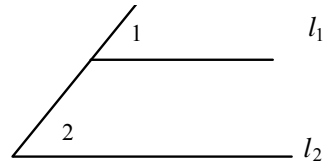


图 3.20

$\angle 1$ 和 $\angle 2$ 为同位角, 若 $\angle 1 = \angle 2$, 则 $l_1 // l_2$

C. 同旁内角互补, 两直线平行

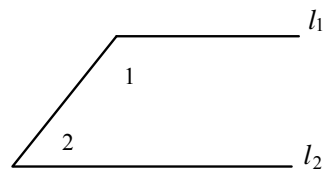


图 3.21

$\angle 1$ 和 $\angle 2$ 为同旁内角, 若 $\angle 1 + \angle 2 = 180^\circ$, 则 $l_1 // l_2$

9. **多边形的一些性质**

A. n 边形内角和为 $(n-2) \times 180^\circ$

B. 正 n 边形各角的度数为

$$\frac{(n-2) \times 180^\circ}{n}$$

C. 正六边形中最长的对角线长度为边长的 2 倍, 若边长为 a , 则对角线长度 = $2a$

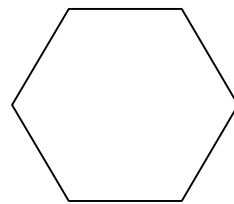


图 3.22

10. **圆与多边形**

Inscribed Polygon in a Circle (圆的内接多边形): 如果一个多边形的所有顶点都有一个圆周上, 那么该多边形称为圆的内接多边形。

Inscribed Circle in a Polygon (圆的外切多边形): 如果一个多边形的每条边都与圆相切, 那么称该多边形为圆的外切多边形, 称该圆为这个多边形的内切圆(the polygon is circumscribed about the circle and the circle is inscribed in the polygon.)

Another common type of GMAT circle problem is a hybrid involving a circle and a square:

(1) A circle with an inscribed square (圆的内接正方形)

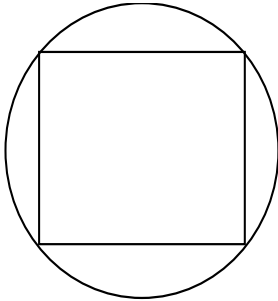


图 3.23

(2) A circle with a circumscribed square(圆的外切正方形)

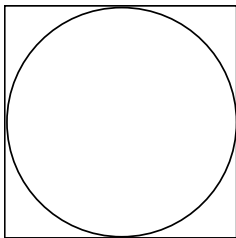


图 3.24

例 1: In the figure below, QRS is a straight line and line TR bisects $\angle PRS$. Is it true that lines TR and PQ are parallel?

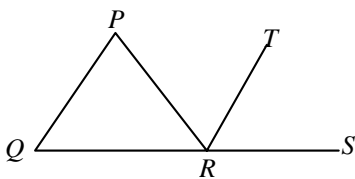


图 3.25

$$(1) PQ = PR \quad (2) QR = PR$$

翻译: 在如图 3.25 所示的图形中, QRS 是一条直线, 并且 TR 平分 $\angle PRS$, TR 与 PQ 平行吗?

解答: 由(1) $PQ = PR$ 无法判断 TR 与 PQ 是否平行; 由(2) $QR = PR$, 则 $\angle PQR$ 与 $\angle QPR$ 相等, 由 TR 平分 $\angle PRS$, 则 $\angle TRS$ 与 $\angle PQR$ 相等, 由同位角相等得到两直线平行。

所以(B) is correct.

3.4.2 立体几何

1. 正方体

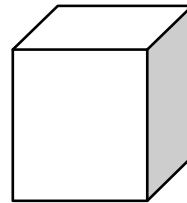


图 3.26

$$\text{表面积} = 6a^2$$

$$\text{体积} = a^3 (a \text{ 为边长})$$

2. 长方体

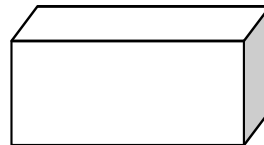


图 3.27

$$\text{表面积} = 2(a \times b + a \times c + b \times c)$$

$$\text{体积} = a \times b \times c (a, b, c \text{ 为长, 宽, 高})$$

3. 圆柱

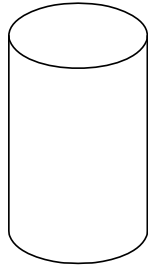


图 3.28

$$\text{表面积} = 2 \times \pi r^2 + 2 \pi r \times h$$

$$\text{体积} = \pi r^2 h (r \text{ 为底边半径, } h \text{ 为高})$$

4. 圆锥

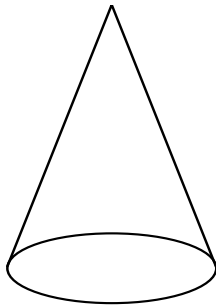


图 3.29

如果圆锥的底面半径是 r , 周长是 c , 侧面母线长是 l , 那么它的侧面积是:

$$S_{\text{侧面积}} = \frac{1}{2} cl = \pi rl$$

它的总表面积是:

$$S_{\text{总}} = S_{\text{侧面积}} + S_{\text{底面积}} = \pi rl + \pi r^2 = \pi r(l+r)$$

如果一个圆锥的底面半径为 r , 高为 h , 那么它的体积是:

$$V_{\text{圆锥}} = \frac{1}{3} \pi r^2 h$$

提示: GMAT 数学很少考到圆锥和球, 即使出现考题, 也一定会把公式在题面给出, 所以这部分知识点只要看一下就可以, 不用深究。

5. 球

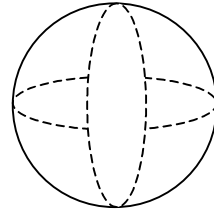


图 3.30

球的表面面积等于它的大圆的面积的 4 倍:

$$S_{\text{球}} = 4 \pi r^2$$

如果球的半径是 r , 那么它的体积是:

$$V_{\text{球}} = \frac{4}{3} \pi r^3$$

例 1: What is the volume of a certain rectangular solid?

(1) Two adjacent faces of the solid have areas 15 and 24, respectively.

(2) Each of two opposite faces of the solid has area 40

翻译: 长方体的体积是多少?

(1) 长方体两个相邻面的面积分别为 15 和 24。

(2) 长方体两个相对面的面积是 40。

解答: 设长方体 3 边边长为 x, y, z , 则长方体的体积为 xyz 。(1)中得到 $xy = 15$, $yz = 24$, 无法得到 xyz 的值, 所以(1)无法单独回答上面的问题; (2)中得到 $xz = 40$, 所以(2)也无法单独回答上面问题; (1)+(2)时, 3 个方程可以求得 x, y, z , 则回答了上面问题。

所以(C) is correct.

提示: 长方体中相邻的 3 个面的面积分别为 x, y, z , 则长方体的体积为:

$$V = \sqrt{xyz}$$

3.4.3 平面直角坐标系

1. Coordinate Signs and the Four Quadrants(坐标平面和四个象限)

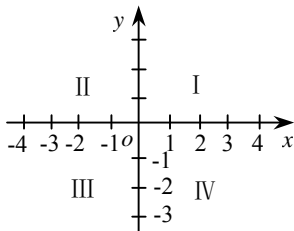


图 3.31

坐标平面的中心,即 x 轴和 y 轴的交点叫做原点,原点的坐标是 $(0, 0)$;任何一个在 x 轴上的点的 y 坐标都是 0,记作 $(x, 0)$;任何一个在 y 轴上的点的 x 坐标都是 0,记作 $(0, y)$ 。

x 和 y 两坐标轴把坐标平面分成四个象限(quadrants),四个象限的相对位置如图所示。一个点的坐标若既不在 x 轴上和 y 轴上,也不是在原点,那么它一定要落在四个象限中的某一个象限中,四个象限中, x 坐标和 y 坐标的符号如下所示:

Quadrant I (+, +)

Quadrant II (-, +)

Quadrant III(-, -)

Quadrant IV(+, -)

2. 坐标平面内的点的对称性

坐标系中若某一点的坐标为 (a, b) ,则此点:

关于直线 $y = x$ 对称的点的坐标为 (b, a) ;

关于直线 $y = -x$ 对称的点的坐标为 $(-b, -a)$;

关于 x 轴对称的点的坐标为 $(a, -b)$;

关于 y 轴对称的点的坐标为 $(-a, b)$;

提示: 若两点关于某条直线对称,则这两点的连线被这条直线垂直平分。

例 1: 若某一点 A 的坐标为 (a, b) ,另一点 B 与 A 的连线被 $y = x$ 这条直线垂直平分, C 点与 B 点的连线被 x 轴垂直平分,问 C 点的坐标?

解答: 由 A, B 两点的连线被 $y = x$ 垂直平分可知 A 和 B 两点一定关于 $y = x$ 这条直线对称,因此 B 点的坐标为 (b, a) ,而 C 点与 B 点的连线又被 x 轴垂直平分,也即 C 点与 B 点关于 x 轴对称,所以 C 点的坐标为 $(b, -a)$ 。

3. 斜率

在坐标系中点 A 为 (a_1, b_1) ,点 B 为 (a_2, b_2) ,则过 A, B 两点的直线的斜率为

$$K = \frac{b_2 - b_1}{a_2 - a_1}$$

4. 截距(intercept)

直线 $y = kx + b$ 与 y 轴相交于点 $B(0, b)$, b 叫做直线 $y = kx + b$ 在 y 轴上的截距,简称截距。

5. 任意两点间的距离

$$P_1P_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

6. 直线方程

(1)点斜式

如果已知某直线的斜率是 k ,通过点 (x_1, y_1) 则该直线的方程:

$$y - y_1 = k(x - x_1)$$

(2)斜截式

如果已知直线的斜率是 k ,在 y 轴上的截距为 b (即与 y 轴的交点是 $(0, b)$),则该直线的方程: $y = kx + b$

这个方程是由直线 l 的斜率和它在 y 轴上的截距确定的,所以叫做直线方程的斜截式。

(3) 两点式

如果直线上的两点坐标已知，分别为 $P_1(x_1, y_1)$ 和 $P_2(x_2, y_2)$ ，则直线的方程为：

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

7. 抛物线方程($y = ax^2 + bx + c, a \neq 0$)

函数 $y = ax^2 + bx + c$ 在坐标系中表现为抛物线 (parabola) 方程，顶点坐标为 $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$ 。

当 $a > 0$ 时，抛物线开口向上，并且 $x \geq -\frac{b}{2a}$ 时，函数为增函数，当 $x \leq -\frac{b}{2a}$ 时，函数为减函数；当 $x = -\frac{b}{2a}$ 时， y 取最小值 $\frac{4ac - b^2}{4a}$ 。

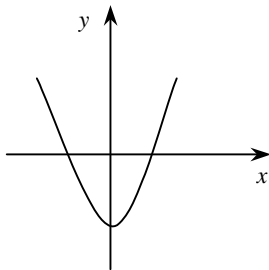


图 3.32

当 $a < 0$ 时，抛物线开口向下，并且 $x \geq -\frac{b}{2a}$ 时，函数为减函数，当 $x \leq -\frac{b}{2a}$ 时，函数为增函数；当 $x = -\frac{b}{2a}$ 时， y 取最大值 $\frac{4ac - b^2}{4a}$ 。

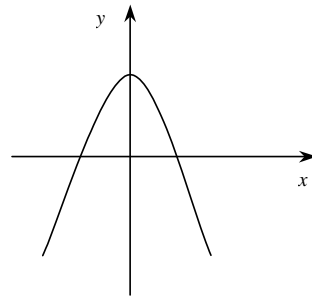


图 3.33

$|c|$ 为抛物线在 y 轴上的截距，当 $c = 0$ 时，抛物线经过原点；当 b 和 c 都为零时，抛物线以原点为顶点。

注：在 GMAT 考试中曾涉及到抛物线 (二次曲线) 在坐标平面上的表达，虽未真正考查抛物线的知识点，但为稳妥起见，还是请参加 GMAT 考试的考生一定要注意抛物线方程的特点。

例 1: In the figure below, segments PR and QR are each parallel to one of the rectangular coordinate axes. Is the ratio of the length of QR to the length of PR equal to 1?

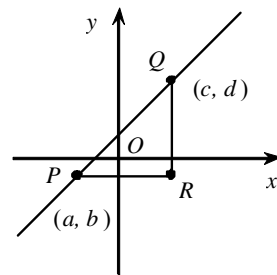


图 3.34

(1) $c = 3$ and $d = 4$

(2) $a = -2$ and $b = -1$

翻译: 在如图 3.34 所示的图形中，线段 PR 和 QR 每个都平行于一条坐标轴，线段 QR 的长度与线段 PR 的长度比率等于 1 吗？

(1) $c = 3$ 并且 $d = 4$

(2) $a = -2$ 并且 $b = -1$

解答: 由 PR 平行于 x 轴而 QR 平行于 y 轴，

则得到 R 点的坐标为 (c, d) , 则线段 PR 的长度为 $c - a$, 线段 QR 的长度为 $d - b$, 由(1)中 $c = 3$ 且 $d = 4$ 无法得到 PR 与 QR 的长度; 由(2)中 $a = -1$ 且 $b = -1$ 也无法得到 PR 与 QR 的长度; 由(1)+(2)可以得到 a, b, c, d 的值所以可以回答这个问题。

所以(C) is correct.

例 2: 正方体, 边长为 4, 问顶点 A 到对面棱的中点 B 连线的长度。

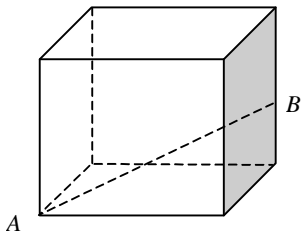


图 3.35

- (A) $4\sqrt{2}$
- (B) $2\sqrt{5}$
- (C) $4\sqrt{3}$
- (D) 6
- (E) 8

解答: $AB = \sqrt{4^2 + 4^2 + 2^2} = 6$

提示: GMAT 数学中的几何问题要求考生能够灵活应用图形的特殊性质。

例 3: 一个正方形 ABCD, 边长 = 1, 外有一点 E, $CE = 1$, $BE = DE$, 求三角形 BCE 面积?

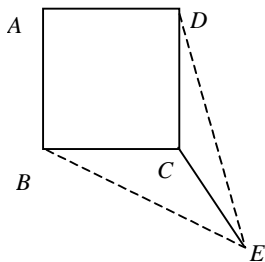


图 3.36

- (A) $\frac{\sqrt{2}}{4}$
- (B) $\frac{\sqrt{2}+1}{4}$
- (C) $\frac{\sqrt{2}}{2}$
- (D) $\frac{1}{4}$
- (E) $\frac{1}{2}$

解答: 由于 $BE = DE$, 则 E 在对角线 BD 的垂直平分线上, 即 E 在 AC 的延长线上。

如图 3.37 所示, 设 AC 和 BD 相交于点 O。则 $S(\triangle BCE) = S(\triangle BOE) - S(\triangle BOC)$

$$\begin{aligned} &= \frac{1}{2} OB \times OE - \frac{1}{2} OB \times OC \\ &= \frac{1}{2} OB \times CE \\ &= \frac{1}{2} \times \frac{\sqrt{2}}{2} \times 1 \\ &= \frac{\sqrt{2}}{4} \end{aligned}$$

所以(A) is correct.

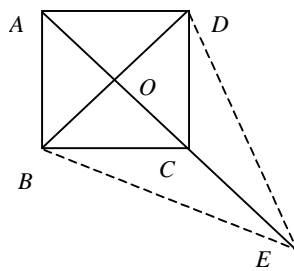


图 3.37

3.5 算术基础习题精选

1. k, m 为连续偶数, 问 $k > m$?

- (1) $k+2$ 同 $m-2$ 不为连续偶数
 (2) $k-1$ 同 $m+3$ 为连续奇数

2. A 参加夏令营, 在除 A 外的 50 人中, 比 A 高或一样高的有 19 人, 比 A 矮或一样矮的有 34 人, 问比 A 矮的人占这 50 人的比例

- (A) 68%
 (B) 62%
 (C) 50%
 (D) 38%
 (E) 30%

3. If x and y are nonzero integers, is $\frac{x}{y}$ an integer?

- (1) x is the product of 2 and some other integer.
 (2) There is only one pair of positive integers whose product equals y .

4. If x is an integer, what is the value of x ?

- (1) $\frac{1}{5} < \frac{1}{x+1} < \frac{1}{2}$
 (2) $(x-3)(x-4) = 0$

5. If K is a positive integer less than 10 and $N = 4,321 + K$, what is the value of K ?

- (1) N is divisible by 3.
 (2) N is divisible by 7.

6. S is a set of integers such that

- i) if a is in S , then $-a$ is in S , and
 ii) if each of a and b is in S , then ab is in S .
 Is -4 in S ?

(1) 1 is in S .

(2) 2 is in S .

7. Is x an even integer?

- (1) x is the square of an integer.
 (2) x is the cube of an integer.

8. If x and y are integers, is $xy + 1$ divisible by 3?

- (1) When x is divided by 3, the remainder is 1.
 (2) When y is divided by 9, the remainder is 8.

9. If \square represents a digit in the 7-digit number $3,62\square,215$, what is the value of \square ?

- (1) The sum of the 7 digits is equal to 4 times an integer.
 (2) The missing digit is different from any of the other digits in the number.

10. Is the positive integer n equal to the square of an integer?

- (1) For every prime number p , if p is a divisor of n , then so is p^2 .
 (2) \sqrt{n} is an integer.

11. An even number x divided by 7 gives some quotient plus a remainder of 6. Which of the following, when added to x , gives a sum which must be divisible by 14?

- (A) 1
 (B) 3
 (C) 7
 (D) 8
 (E) 13

12. If x is the product of three consecutive positive integers, which of the following must be true?
- I. x is an integer multiple of 3.
 - II. x is an integer multiple of 4.
 - III. x is an integer multiple of 6.
- (A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II, and III
13. How many three-digit numerals begin with a digit that represents a prime number and end with a digit that represents a prime number?
- (A) 16
(B) 80
(C) 160
(D) 180
(E) 240
14. If x , y , and z are single-digit integers and $100(x)+1,000(y)+10(z)=N$, what is the units' digit of the number N ?
- (A) 0
(B) 1
(C) x
(D) y
(E) z
15. What is the 1st term in sequence S ?
- (1) The 3rd term in S is 2.
 - (2) The 2nd term in S is twice the 1st, and the 3rd term is three times the 2nd.
16. If y is an odd integer, which of the following must be an even integer?
- (A) $y+2$
(B) $y+6$
(C) $2y-1$
(D) $3y$
(E) $3y+1$
17. All of the following have the same number of distinct prime factors EXCEPT _____.
- (A) 20
(B) 21
(C) 24
(D) 30
(E) 45
18. How many integers between 100 and 150, inclusive, can be evenly divided by neither 3 nor 5?
- (A) 33
(B) 28
(C) 27
(D) 25
(E) 24
19. In a certain chess club, 16 members played in tournament X , 20 played in tournament Y , and 12 played in both tournaments. If each member of the club played in at least one of these two tournaments, how many members does the club have?
- (A) 48
(B) 36
(C) 32
(D) 28

(E) 24

20. If $r=2^3 \cdot 5^2 \cdot 7$ and $s=2^2 \cdot 3^2 \cdot 5$, which of the following is equal to the greatest common divisor of r and s ?

- (A) $2 \cdot 5$
 (B) $2^2 \cdot 5$
 (C) $2^3 \cdot 5^2$
 (D) $2 \cdot 3 \cdot 5 \cdot 7$
 (E) $2^3 \cdot 3^2 \cdot 5^2 \cdot 7$

21. If x is an even integer and y is an odd integer, which of the following is an odd integer?

- (A) $x(y-1)$
 (B) $xy+2$
 (C) $2(x+y)$
 (D) $2x+y$
 (E) $x+2y$

参考答案

- 1~5 DBECB
 6~10 BECCB
 11~15 DDCAC
 16~20 EDCEB
 21 D

3.6 排列组合概率习题精选

- 10 人中有 6 人是男性, 问组成 4 人组, 3 男 1 女的组合数。
- 4 对人, 任取 3 人, 组成一个小组, 不能从任意一对中取 2 个, 问有多少种可能性?
- 15 人中取 5 人, 有 3 个不能都取, 有多少种取法?
- 7 人比赛, A 在 B 的前面的可能性有多少种?
- 3 对人分为 A,B,C 三组, 考虑组的顺序和组中的人的顺序, 有多少种分法?
- 17 个人中任取 3 人分别放在 3 个屋中, 其中 7 个只能在某两个屋, 另外 10 个只能在另一个屋, 有多少种分法?
- A,B,C,D,E,F 排在 1, 2, 3, 4, 5, 6 这六个位置, 问 A 不在 1, B 不在 2, C 不在 3 的排列的种数?
- 4 幅大小不同的画, 要求两幅最大的排在一起, 有多少种排法?
- 5 辆车排成一排, 1 辆黄色, 1 辆蓝色, 3 辆红色, 且 3 辆红车不可分辨, 问有多少种排法?
- 6 个身高不同的人分成 2 排, 每排 3 人, 每排从左到右, 由低到高, 且后排的人比他身前的人高, 问有多少种排法?
- 一个均匀硬币 $2N$ 次, 求出现正面 K 次的概率。
- 有 5 个白色珠子和 4 个黑色珠子, 从中任取 3 个, 问其中至少有一个是黑色的概率。
- 自然数集合 S 中所有满足 $n \leq 100$, 若 $n(n+1)(n+2)$ 能够被 6 整除, 求 n 取值的概率?
- 设 O 为正方形 ABCD [坐标为 $(1, 1)$, $(1, -1)$, $(-1, 1)$, $(-1, -1)$] 中的一点 (x, y) , 求满足 $x^2+y^2 < 1$ 的概率。

15. $A > B$ (成功的概率)?
- (1) A 半部分的成功概率为 1%, B 前半部分成功概率为 1.4%。
- (2) A 后半部分的成功概率为 10%, B 后半部分成功概率为 8.5%。
16. 集合 A 中有 100 个数, B 中有 50 个数, 并且满足 A 中元素于 B 中元素关系 $a+b=10$ 的有 20 对。问任意分别从 A 和 B 中各抽签一个, 抽到满足 $a+b=10$ 的 a, b 的概率。
17. 有两组数, 都是 $[1, 2, 3, 4, 5, 6]$, 分别任意取出两个, 其中一个比另一个大 2 的概率?
18. 从 0 到 9 这 10 个数中任取一个数并且记下它的值, 再取一个数也记下它的值。当两个值的和为 8 时, 出现 5 的概率是多少?
19. 5 双不同颜色的袜子, 从中任取两只, 是一对的概率为多少?
20. 从 0 到 9 中挑出 4 个数编 4 位数的电话号码, 求首位不是 0 且数字不重复的概率。
21. 两把 keys, 放到有 5 个 keys 的 keychain(直线)中, 相邻的概率为多少?
22. 3 男生, 3 女生, 从中挑出 4 个, 问男女相等的概率?
23. 4 对夫妇, 从中任意选出 3 人组成一个小组, 不能从任一对夫妇中同时选择两人, 问符合选择条件的概率是多少?
24. 从 6 双不同的手套中任取 4 只, 求其中恰有一双配对的概率。
25. 3 个打字员为 4 家公司服务, 每家公司各有一份文件录入, 问每个打字员都收到文件的概率?
26. 有 4 组人, 每组一男一女, 从每组各取一人, 问取出两男两女的概率。
27. 一个人掷飞标, 其中击中靶心的概率为 0.7, 他连续掷 4 次飞标, 有 2 次击中靶心的概率为多少?
28. 某种硬币每掷一次正面朝上的几率为 0.6, 问连续抛 5 次, 至少有 4 次朝上的概率。
29. A 的发生概率为 0.6, B 发生的概率为 0.5, 问 A, B 都不发生的最大概率?
30. 某种动物由出生而活到 20 岁的概率为 0.7, 活到 25 岁的概率为 0.56, 求现龄为 20 岁的这种动物活到 25 岁的概率。
31. 一只袋中状语 5 个乒乓球, 其中 3 只白色, 2 只红色, 现从袋中取球两次, 每次一只, 取出后不足放回。
- 试求: (1) 两只球都是白色的概率。
(2) 两只球颜色不同的概率。
(3) 至少有一只白球的概率。
32. 甲乙两个射手彼此独立地射击同一目标各一次, 甲射中的概率为 0.9, 乙射中的概率为 0.8, 求目标被射中的概率。
33. 三人独立地去破译一个密码, 他们能译出的概率分别为 $1/5, 1/3, 1/4$, 求将此密码译出的概率。
34. 某市共有 10000 辆自行车, 其牌照号码从 00001 到 10000, 求偶然遇到的一辆自行车, 其牌照号码中有数字 8 的概率。
35. 电话号码由四个数字组成, 每个数字可以是 0、1、2、3、4、5、6、7、8、9 中的任一个数, 求电话号码是由完全不同的数字组成的概率。
36. 袋中有 a 只白球, b 只红球, 依次将球一只只摸出, 不放回, 求第 K 次摸出白球的概率 ($1 < k < a+b$)
37. 3 封不同的信, 有 4 个信箱可供投递, 共有多少种投信的方法?
38. 有 5 个队伍参加了某联赛, 两两之间进行循环赛两场, 没有平局, 试问总共轮的场次是多少?

39. 从5位男同学和4位女同学中选出4位参加一个座谈会,要求与会成员中既有男同学又有女同学,有几种不同的选法?
(D) 4
(E) 3
40. 七人并坐,甲不坐首位,乙不坐末位,有几种不同的坐法?
41. 用0、2、4、6、9这五个数字可以组成数字不重复的五位偶数共有多少个?
42. 6张同排联号的电影票,分给3名男生和3名女生,如欲男女相间而坐,则不同的分法数为多少?
43. 甲乙丙丁戊五人并排站成一排,如果乙必须站在甲的右边(甲乙可以不相邻),那么不同的排法共有多少种?
44. 晚会上有5个不同的唱歌节目和3个不同的舞蹈节目,问:分别按以下要求各可排出几种不同的节目单?
(1) 3个舞蹈节目排在一起
(2) 3个舞蹈节目彼此分隔
(3) 3个舞蹈节目先后顺序一定
45. 几本不同的书分给2人,每人2本,不同的分法有多少种?
46. 12支铅笔,6红6蓝,取出四红不放回,求第五支为红的概率
47. 200辆车,其中既不为黑也不是轿车的有多少?
(1) $\frac{2}{5}$ 为黑, $\frac{3}{4}$ 为轿车
(2) $\frac{3}{10}$ 为黑色轿车
48. Three stacks containing equal numbers of chips are to be made from 9 red chips, 7 blue chips, and 5 green chips. If all of these chips are used and each stack contains at least 1 chip of each color, what is the maximum number of red chips in any one stack?
(A) 7
(B) 6
(C) 5
49. There are 6 groups in a room. Each group consists of 3 men. How many handshakes will there be if each man only shakes hands with people who are outside his group?
50. if you tossed a coin three times, what's the probability that you get the same side all three times.

参考答案

1. $C_6^3 C_4^1$

2. $C_8^3 - C_4^1 C_6^1$ 先取得所有的组合数, 然后减去选取了成对的情况,

3. $C_{15}^5 - C_{12}^2$

4. $P_7^7 / 2$ A 在 B 前的次数与在其后的次数相等

5. $P_3^3 \times (P_2^2)^3$ 先考虑组顺, 再考虑人顺

6. $P_7^2 P_{10}^1$

7. $P_6^6 - 3P_5^5 + 3P_4^4 - P_3^3$ (先取总数, 后分别把 A 放 1, B 放 2, C 放 3, 把这个数量算出, 从总数中减去即可, 建议用三个同样的环相互交错取总数的方法计算)

8. $2P_3^3$

9. P_5^5 / P_3^3 如果再加一个条件 2 辆不可分辨的白色车, 同理: $P_7^7 / P_3^3 P_2^2$

10. 5 种。穷举法。6 个人, 为 1、2、3、4、5、6, 即

1		
	5	6

1, 5, 6, 三数固定, 把 2, 3, 4, 在里面摆。

11. $\frac{C_{2n}^k}{2^{2n}}$ 独立重复试验。如果在一次试验中

某事件发生的概率是 P , 那么在 n 次独立重复试验中这个事件恰好发生 K 次的概率为 $P_n(K) = C_n^k P^k (1-P)^{n-k}$

一夫妇生四孩子, 问生 2 男 2 女的情况

之几率; 每次生男女概率相同, $1/2$, 如抛硬币问题(抛四次, 2 次朝上), 即 $C_4^2 (1/2)^4 = 3/8$

12. $1 - C_5^3 / C_9^3$

13. 由于 3 个连续自然数必包括一个偶数及一个可被 3 整除的数, 因此概率是 100%。

14. 面积法。 $x^2 + y^2 = 1$ 为一个以原点为圆心, 半径为 1 的圆, 面积为 π , 正方形面积为 4, ANSWER: $\pi/4$

15. $P(A) = 1\% \times 10\%$

$P(B) = 1.4\% \times 8.5\%$

16. $C_{20}^1 / C_{100}^1 C_{50}^1$

17. $2 \times 4 / C_6^1$ C_6^1 分两次取。

18. $2/9$. 总共有 $\{(8, 0)(0, 8)(1, 7)(7, 1)(6, 2)(2, 6)(5, 3)(3, 5)(4, 4)\}$ 集合中不能有重复元素

19. $5 / C_{10}^2$

20. $(P_{10}^4 - C_9^3) / 10^4$

21. 两把 keys 放入后的排列为 P_7^2 , 两把 keys 相邻的情况把两把看成一把, 再排两把 keys, 即再 $\times 2$, 所以为 $2C_6^1 / P_7^2$ 。放入环的情况相当于放入 4 个 keys 的直线中, $2C_5^1 / P_6^2$ 。

22. $C_3^2 C_3^2 / C_6^4$

23. $(C_8^3 - C_6^1 C_4^1) / C_8^3$

24. $C_6^1 C_5^2 C_2^1 C_2^1 / C_{12}^4$

25. $(C_4^2 C_2^1) C_3^1 / 3^4$ 先把文件分为 2、1、1 三堆, 然后把这三堆文件分给三个打字

员。

$$26. C_4^2(1/2)^4 = 3/8$$

$$27. C_4^2 \times 0.7^2 \times 0.3^2$$

$$28. 0.6^5 + C_5^4 \times 0.6^4 \times 0.4$$

29. 0.4 请画两个圆分别代表 A 、 B 发生的概率，当 B 包含于 A 时，即是 A 、 B 都不发生的最大概率。

$$I=A+B-A \quad B+AB$$

$$AB=1-0.6-0.5+0.6 \quad 0.5=0.4$$

$$30. 0.56/0.7$$

$$P(0-20) \times P(20-25) = P(0-25)$$

$$31. (1) C_3^2 / C_5^2$$

$$(2) C_3^1 C_2^1 / C_5^2$$

$$(3) 1 - C_2^2 / C_5^2$$

$$32. 1-(1-0.9)(1-0.8)=0.98$$

$$33. 1-(1-1/5)(1-1/3)(1-1/4)=3/5$$

$$34. 1 - \frac{9^4 - 1 + 1}{10000}$$

$$35. \frac{P_{10}^4}{10^4}$$

$$36. \frac{a}{a+b}$$

$$37. 4^3$$

$$38. 2C_5^2$$

$$39. C_9^4 - C_5^4 - C_4^4 = 120$$

$$40. P_7^7 - 2P_6^6 + P_5^5 = 3720$$

$$41. P_5^5 - 2P_4^4 + P_3^3 = 78$$

$$42. 2P_3^3 P_3^3 = 72$$

$$43. \frac{1}{2} P_5^5$$

44. (1) $P_3^3 P_6^6$ 先把 3 个舞蹈节目看成一个节目与唱歌节目进行排列，后内部排列

$$(2) C_6^3 P_3^3 P_5^5$$

$$(3) P_8^5$$

$$45. C_4^2 \quad (3 \text{ 本分给 2 人分法: } 2C_3^1)$$

(6 本分给 3 人，每人 2 本分法: $C_6^2 C_4^2$)

$$46. \frac{6}{12} \cdot \frac{5}{11} \cdot \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{2}{8} = \frac{1}{132}$$

$$47. (1) \text{ 最多为 } 50$$

$$(2) \text{ 最多为 } 140$$

$$48. C$$

$$49. 135 = \frac{45 \times 6}{2}$$

$$50. \frac{1}{8} \times 2 = \frac{1}{4}$$

3.7 代数部分习题精选

1. $101 \leq n \leq 150$, 求 $1/n$ 的总和的范围

- (A) $1/3 < 1/n < 1/2$
 (B) $1/6 < 1/n < 1/5$
 (C) $1/15 < 1/n < 1/12$
 (D) $1/30 < 1/n < 1/20$
 (E) $2 < 1/n < 3$

2. Is $x - y > r - s$?

- (1) $x > r$ and $y > s$?
 (2) $y = 2, s = 3, r = 5$, and $x = 6$.

3. If n is an integer between 2 and 100 and if n is also the square of an integer, what is the value of n ?

- (1) n is the cube of an integer.
 (2) n is even.

4. Is $x^2 - y^2$ a positive number?

- (1) $x - y$ is a positive number.
 (2) $x + y$ is a positive number.

5. A certain company paid bonuses of \$125 to each of its executive employees and \$75 to each of its nonexecutive employees. If 100 of the employees were nonexecutives, how many were executives?

- (1) The company has a total of 120 employees.
 (2) The total amount that the company paid in bonuses to its employees was \$10,000.

6. For integers a, b , and c , $\frac{a}{b-c} = 1$, What is the value of $\frac{b-c}{b}$?

- (1) $\frac{a}{b} = \frac{3}{5}$
 (2) a and b have no common factors greater

than 1.

7. If a, b , and c are integers, is the number $3(a+b) - c$ divisible by 3?

- (1) $a + b$ is divisible by 3.
 (2) c is divisible by 3.

8. If $y \neq 0$ and $y \neq -1$, which is greater,

$\frac{x}{y}$ or $\frac{x}{y+1}$?

- (1) $x \neq 0$
 (2) $x > y$

9. If x and y are consecutive odd integers, what is the sum of x and y ?

- (1) The product of x and y is negative.
 (2) One of the integers is equal to -1 .

10. If $x \neq 0$, is $|x| < 1$?

- (1) $x^2 < 1$
 (2) $|x| < \frac{1}{x}$

11. What is the value of $k^2 - k$?

- (1) The value of $k - \frac{1}{k}$ is 1.
 (2) The value of $2k - 1$ is $\sqrt{5}$.

12. If $(16)(3)^2 = x(2^3)$, then $x =$

- (A) 81
 (B) 72
 (C) 18
 (D) 16
 (E) 8

13. $(165)^2 - (164)^2 =$

- (A) 1
 (B) 2
 (C) 4

- (D) 325
(E) 329
14. If x , y and z are non-zero numbers such that $1 \geq y > x$ and $xy = z$, which of the following cannot be true?
(A) $y > z$
(B) $y = z$
(C) $z = x$
(D) $x > z$
(E) $z > 0$
15. The function $*$ is defined by the equation $a * b = \frac{ab}{b-a}$, where $a \neq b$. Which of the following has a value of 3?
(A) $1 * 3$
(B) $3 * 0$
(C) $2 * 6$
(D) $6 * 2$
(E) $4 * -1$
16. If $20,000 = 5^n \cdot 2^{n+1}$, then $n =$
(A) 2
(B) 3
(C) 4
(D) 5
(E) 6
17. How many factors of 60 are greater than $\sqrt{60}$?
(A) Twelve
(B) Nine
(C) Eight
(D) Seven
(E) Six
18. On a certain 10-question test, each question after the first question is worth 2 points more than the previous question. If the greatest number of points that can be scored on the test is 100, how many points is the eighth question worth?
(A) 9
(B) 14
(C) 15
(D) 19
(E) 33
19. Which of the following CANNOT be zero?
I. The sum of 7 consecutive integers
II. The sum of 10 consecutive even integers
III. The product of 13 consecutive integers
(A) II only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III
20. A computer can perform 1,000,000 calculations per second. At this rate, how many hours will it take this computer to perform the 3.6×10^{11} calculations required to solve a certain problem?
(A) 60
(B) 100
(C) 600
(D) 1,000
(E) 6,000
21. Which of the following inequalities is an algebraic expression for the shaded part of the number line below?

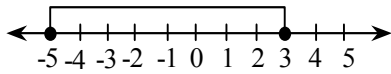


图 3.38

- (A) $|x| \leq 3$
 (B) $|x| \leq 5$
 (C) $|x-2| \leq 3$
 (D) $|x-1| \leq 4$
 (E) $|x+1| \leq 4$

参考答案:

- | | |
|-------|-------|
| 1~5 | ABACD |
| 6~10 | ABEAD |
| 11~15 | DCEBC |
| 16~20 | CECAB |
| 21 | E |

3.8 几何部分习题精选

1. If the Longfellow Playground is rectangular, what is its width?

- (1) The ratio of its length to its width is 7 to 2.
 (2) The perimeter of the playground is 396 meters.

2. In parallelogram $ABCD$ below, what is the measure of $\angle ADC$?

- (1) The measure of $\angle ABC$ is greater than 90° .
 (2) The measure of $\angle BCD$ is 70° .

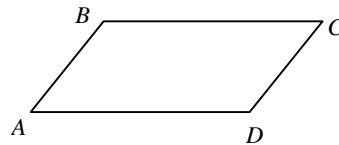


图 3.39

3. In $\triangle JKL$ shown below, what is the length of segment JL ?

- (1) $JK = 10$
 (2) $KL = 5$

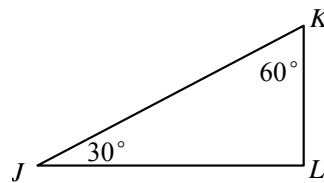


图 3.40

4. The figure below represents the floor of a square foyer with a circular rug partially covering the floor and extending to the outer edges of the floor as shown. What is the area of the foyer that is not covered by the rug?

- (1) The area of the foyer is 9 square

meters.

- (2) The area of the rug is 2.25π square meters.

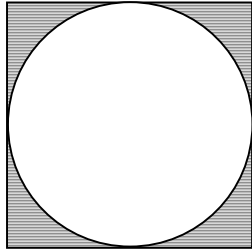


图 3.41

5. Is quadrilateral Q a square?

- (1) The sides of Q have the same length.
 (2) The diagonals of Q have the same length.

6. The surface area of a square tabletop was changed so that one of the dimensions was reduced by 1 inch and the other dimension was increased by 2 inches. What was the surface area before these changes were made?

- (1) After the changes were made, the surface area was 70 square inches.
 (2) There was a 25 percent increase in one of the dimensions.

7. If l_1, l_2 and l_3 are lines in a plane, is l_1 perpendicular to l_3 ?

- (1) l_1 is perpendicular to l_2 .
 (2) l_2 is perpendicular to l_3 .

8. A rectangular floor that is 4 meters wide is to be completely covered with nonoverlapping square tiles, each with side of length 0.25 meter, with no portion of any tile remaining. What is the least number of such tiles that will be required?

- (1) The length of the floor is three times the width.
 (2) The area of the floor is 48 square meters.

9. In the figure below, D is a point on side AC of $\triangle ABC$. Is $\triangle ABC$ isosceles?

- (1) The area of triangular region ABD is equal to the area of triangular region DBC .
 (2) $BD \perp AC$ and $AD = DC$

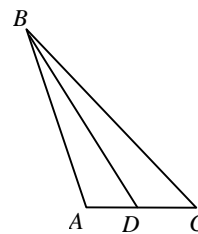


图 3.42

10. If the area of triangular region RST is 25, what is the perimeter of RST ?

- (1) The length of one side of RST is $5\sqrt{2}$.
 (2) RST is a right isosceles triangle.

11. Rectangle $ABCD$ is inscribed in a circle as shown below. What is the radius of the circle?

- (1) The length of the rectangle is $\sqrt{3}$ and the width of the rectangle is 1.
 (2) The length of arc AB is $\frac{1}{3}$ of the circumference of the circle.

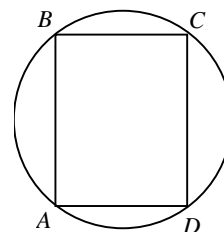


图 3.43

12. In the figure below, line AC represents a seesaw that is touching level ground at point A . If B is the midpoint of AC , how far above the ground is point C ?

- (1) $x = 30$.
 (2) Point B is 5 feet above the ground.

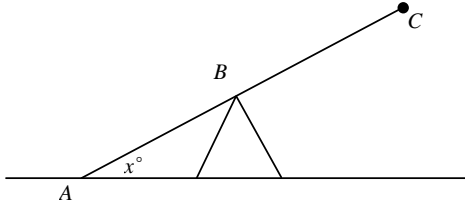


图 3.44

13. 正方体，边长为 4，问顶点 A 到顶点 B 连线的长度。

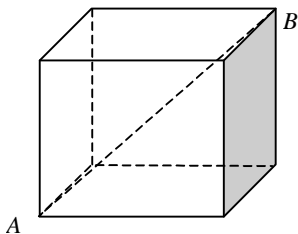


图 3.45

- (A) $4\sqrt{2}$
 (B) $2\sqrt{5}$
 (C) $4\sqrt{3}$
 (D) 6
 (E) 8

14. Is $\triangle RST$ a right triangle?

- (1) The degree measure of $\angle R$ is twice the degree measure of $\angle T$.
 (2) The degree measure of $\angle T$ is 30.

15. What is the volume of a certain cube?

- (1) The sum of the areas of the faces of the cube is 54.
 (2) The greatest possible distance between two points on the cube is $3\sqrt{3}$.

16. The number of diagonals of a polygon of n Sides is given by the formula $d = n(n-3)$.

If a polygon has twice as many diagonals as sides, how many sides does it have?

- (A) 3
 (B) 5
 (C) 6
 (D) 7
 (E) 8

17. A rectangle with dimensions 24 inches by 42 inches is to be divided into squares of equal size; Which of the following could be a length of a side of the squares?

- (A) 4 inches
 (B) 6 inches
 (C) 7 inches
 (D) 8 inches
 (E) 10 inches

18. In the rectangular coordinate system below, the shaded region is bounded by straight lines. Which of the following is NOT an equation of one of the boundary lines?

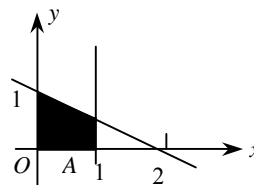
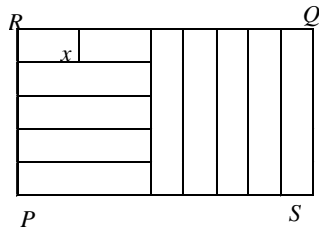


图 3.46

- (A) $x=0$
- (B) $y=0$
- (C) $x=1$
- (D) $x-y=0$
- (E) $x+2y=2$

19. Rectangular region $PQRS$ shown below is partitioned into ten identical smaller rectangular regions, each of which has width x . What is the perimeter of $PQRS$ in terms of x ?



- (A) $15x$
- (B) $25x$
- (C) $30x$
- (D) $50x$
- (E) It cannot be determined from the information given.

20. What is the circumference of the circle below?

- (1) The length of arc XYZ is 18.
- (2) $r = s$.

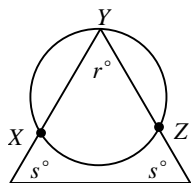


图 3.48

21. What is the radius of the circle below with

center O ?

- (1) The ratio of OP to PQ is 1 to 2.
- (2) P is the midpoint of chord AB .

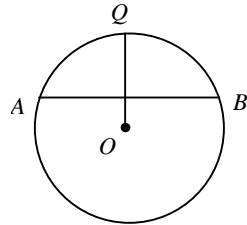


图 3.49

22. If $ab \neq 0$, in what quadrant of the coordinate system below does point (a, b) lie?

- (1) (b, a) lies in quadrant IV.
- (2) $(a, -b)$ lies in quadrant III.

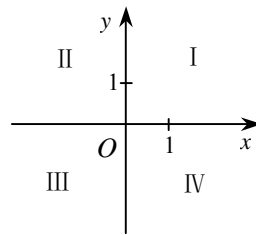


图 3.50

23. In the figure below, segments RS and TU represent two positions of the same ladder leaning against the side SV of a wall. The length of TV is how much greater than the length of RV ?

- (1) The length of TU is 10 meters.
- (2) The length of RV is 5 meters.

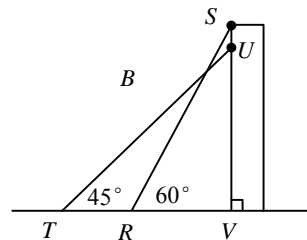


图 3.51

24. In the figure above, what is the measure of $\angle ABC$?

- (1) BX bisects $\angle ABY$ and BY bisects $\angle XBC$.
- (2) The measure of $\angle ABX$ is 40° .

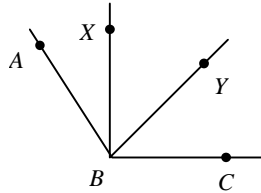


图 3.52

25. The inside of a rectangular carton is 48 centimeters long, 32 centimeters wide, and 15 centimeters high. The carton is filled to capacity with k identical cylindrical cans of fruit that stand upright in rows and columns, as indicated in the figure above. If the cans are 15 centimeters high, what is the value of k ?

- (1) Each of the cans has a radius of 4 centimeters.
- (2) 6 of the cans fit exactly along the length of the carton..

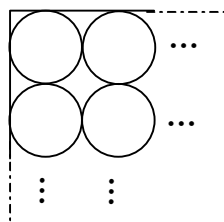


图 3.53

26. In the figure below, what is the area of the circular region with center O ?

- (1) MN is perpendicular to RS .

(2) The area of triangular region OMT is 4.

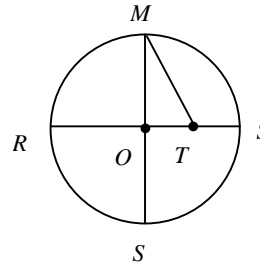


图 3.54

27. In the figure above, if PR is a line segment, what is the sum of the lengths of the curved paths from P to Q and from Q to R ?

- (1) $XQ = QY = 5$ centimeters.
- (2) Every point on arc PQ is 5 centimeters from point X , and every point on arc QR is 5 centimeters from point Y .

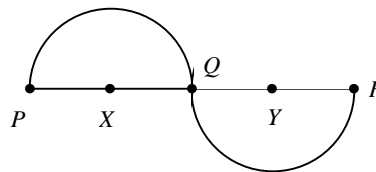


图 3.55

参考答案:

- | | |
|-------|-------|
| 1~5 | CBDDC |
| 6~10 | CCDBB |
| 11~15 | ABCCD |
| 16~20 | BBDCC |
| 21~25 | EDDCD |
| 26~27 | EB |

第4章 理解错误

英文理解的错误也是制约很多考生取得高分的一个重要原因。例如很多考生认为 decrease to 和 decrease by 是同样的意思。有些数学术语对很多考生来说很陌生。如 denominator, acute angle 等。

还有一些考生不熟悉图表问题，无法将题目，问题和图表联系起来，会在图表题上耗费很多时间。

英文理解能力差，对数学术语不熟悉，对图表题很陌生，这些都是考生取得高分的巨大障碍。本章即是针对这些问题展开的。

4.1 英文理解能力差

许多考生因为英文理解能力差，在 GMAT 数学考试中不能取得高分。由于题目本身用英文叙述，若阅读能力欠佳，则无法掌握问题的核心所在，即使数学能力很强，也无法创造佳绩。

部分考生也会遇到生僻单词，但绝大多数单词其实并不影响考生答题，不要花太多时间去猜测单词的意义。GMAT 数学最关键的还是理清题目的头绪，对题目中的概念做概括或者替换，将文字叙述转化为数学问题。

注意：部分难度较大的代数题目涉及到如何把一个 verbal expression 变为一个 algebraic expression 的问题，例如：

Verbal Expression	Algebraic expression
Fourteen more than x	$x+14$
Six less than twice x	$2x-6$
The square of the sum of x and 9	$(x+9)^2$
The sum of the square of x and the square of 9	x^2+9^2
The distance traveled by a car going 60 miles an hour for t hours	$60t$
The average of 80, 88, 90, and x	$\frac{80+88+90+x}{4}$

例 1: A certain clock marks every hour by striking a number of times equal to the hour and the time required for a stroke is exactly equal to the time interval between strokes. At 6:00 the time lapse between the beginning of the first stroke and the end of the last stroke is 22 seconds. At 12:00, how many seconds elapse between the beginning of the first stroke and the end of the last stroke?

- (A) 72
- (B) 50
- (C) 48
- (D) 46
- (E) 44

翻译: 某一闹钟整点时敲击的次数等于那一小时的钟点数，一次敲击所持续的时间等于再次敲击之间的间隔时间。在 6:00，第一次敲击与最后一次敲击之间所间隔的时间为 22 秒。在中午 12:00，

在第一次敲击和最后一次敲击中间间隔的秒数是多少?

解答: 本题意为敲击持续时间与两次敲击间隔时间相同, 设该时间为 x 秒, 注意间隔的次数比敲击的次数少 1 次, 则 6:00

共敲击 6 次间隔为 5 次, 则得到如下方程:

$$(6 + 5)x = 22 \quad x = 2$$

则 12:00 的敲击与间隔时间共为 $(12 + 11) \times 2 = 46$ 秒

所以(D) is correct.

错误分析: 本题的关键是理解 “the time required for a stroke is exactly equal to the time interval between strokes”。考生同时应该注意到 n 次敲击有 $n-1$ 次间隔次数。

例 2: A rectangle is defined to be “silver” if and only if the ration of its length to its width is 2 to 1. If rectangle S is silver, is rectangle R silver?

(1) R has the same area as S.

(2) The ratio of one side of R to one side of S is 2 to 1.

翻译: 一个长方形定义为 “silver” 假如并且仅仅假如其长和宽的比率为 2:1, 假如一个长方形 S 是 “silver”, 长方形 R 是 “silver” 吗?

(1) R 与 S 面积相同

(2) R 的一条边与 S 的一条边的比率是 2:1。

解答: 由(1)无法得到任何结论; 由(2)不知道另一条边的比率是多少, 所以也无法回答上面的问题; (1)+(2)可以得到两个结果, 所以也无法回答上面的问题。

所以(E) is correct.

错误分析: 有的考生地将 “The ratio of one side of R to one side of S is 2 to 1.” 理解

为“R 的每一条边与 S 的每一条边的比率是 2:1”, 从而地选择了选项 B。

例 3: The inflation index for the year 1989 relative to the year 1970 was 3.56, indicating that, on the average, for each dollar spent in 1970 for goods, \$3.56 had to be spent for the same goods in 1989. If the price of a Model K mixer increased precisely according to the inflation index, what was the price of the mixer in 1970?

(1) The price of the Model K mixer was \$102.40 more in 1989 than in 1970.

(2) The price of the Model K mixer was \$142.40 in 1989.

翻译: 1989 年相对于 1970 年的通货膨胀的指数是 3.56, 它显示出: 平均而言, 1970 年花费在商品上的每一美元是 1989 年花费在相同商品上的 3.56 美元。假如 K 牌混和器的价格正好增加了通货膨胀的指数, 该混和器在 1970 的价格是多少?

(1) K 牌混和器 1989 年的价格比 1970 年的价格高 102.4 美元。

(2) K 牌混和器的价格在 1989 年是 142.40 美元。

解答: 设该混和器在 1970 年的价格为 x 美元, 由(1) $x + 102.4 = 3.56x$ 可以求得 x 的值; 由(2)得 $x = 142.40 \div 3.56$ 也可以求得 x 的值
所以(D) is correct.

错误分析: 有的考生不认识 “inflation index”, 还是花了大量时间去思考。其实毫无必要, 因为题目随即有对 “inflation index” 的解释。

例 4: Last year the annual premium on a certain hospitalization insurance policy

was \$ 408, and the policy paid 80 percent of any hospital expenses incurred. if the amount paid by the insurance policy last year was equal to the annual premium plus the amount of hospital expenses not paid by the policy, what was the total amount of hospital expenses last year?

- (A) \$850.00
- (B) \$680.00
- (C) \$640.00
- (D) \$510.00
- (E) \$326.40

翻译: 去年, 某一医院保险政策的年保险费是 408 美元, 而且保险政策付给医院 80% 的保险金。假如去年保险政策所付的钱与年保险费和保险政策未付的医疗费用和相等, 去年的医疗费用是多少?

解答: 设去年的医疗费用为 x 美元, 则:

$$408 + x \times 20\% = x \times 80\%$$

$$x = 680 \text{ 美元}$$

所以(B)is correct。

错误分析: 本题中有两个概念: hospital expenses 和 annual premium, 而 hospital expenses 又分为两部分: paid 和 not paid。根据题意可列方程求解。

提示: 如果一个题目涉及到多个概念, 解题的关键是理清题目的头绪, 对题目中的概念做概括或者替换。

如本题可以替换为 “hospital expenses 有 80% paid by the policy, 20% not paid。若已知 policy 为 \$408, 且 hospital expenses paid by the policy 等于 policy 和 not paid 之和。问 hospital expenses 是多少?”

例 5: If Carmen had 12 more tapes, she would have twice as many tapes as Rafael. Does Carmen have fewer tapes than Rafael?

- (1) Rafael has more than 5 tapes.
- (2) Carmen has fewer than 12 tapes.

翻译: 假如 C 再多 12 盘磁带, 她可能是 R 磁带量的两倍, C 比 R 的磁带少吗?

- (1) R 有超过 5 盘磁带
- (2) C 有少于 12 盘磁带

解答: 由题可得:

$$\begin{aligned} C + 12 &= 2R \\ C - R &= R - 12 = \frac{C - 12}{2} \end{aligned}$$

只有 C 和 12 或者 R 和 12 的关系, 才能够回答以上问题。

由(1)无法决定是 C 的磁带多还是 R 的磁带多; 由(2) $C < 12$, 则进化论怎样取值都要满足上面等式 $R > C$, 例如 $C = 10$, $R = 11$, 所以, (2)单独回答了上面问题, 所以(B)is correct。

提示: 注意理解本题中的虚拟语气 “had...would have...”, 即如果 C 再多 12 盘, 则是 R 的两倍。

例 6: An infinite sequence of positive integers is called an “alpha sequence” if the number of even integers in the sequence is finite. If S is an infinite sequence of positive integers, is S an alpha sequence?

- (1) The first ten integers in S are even.
- (2) An infinite number of integers in S are odd.

翻译: 一个无限的正整数数列被称为 α 数列, 假如数列中的偶数的数目是有限的。假如 S 是一个无限的正整数的数列, S 是一个 α 数列吗?

- (1) S 中的前 10 个整数是偶数
- (2) S 中的奇数的数目是无限的

解答： α 数列的定义为：其中的偶数数目是有限的。由(1)无法得知偶数数目是否有限；由(2)中奇数数目无限，偶数数目也可能无限；由(1)+(2)也无法得到任何结果。

所以(E) is correct。

错误分析：本题实际上等效于问“S 数列中偶数的数目是有限的吗？”，和 α 数列并没有多少关系。

例 7：There were 36,000 hardback copies of a certain novel sold before the paperback version was issued. From the time the first paperback copy was sold until the last copy of the novel was sold, 9 times as many paperback copies as hardback copies were sold. If a total of 441,000 copies of the novel were sold in all, how many paperback copies were sold?

- (A) 45,000
- (B) 360,000
- (C) 364,500
- (D) 392,000
- (E) 396,900

翻译：某一小说在简装本出售前有 36000 本精装本被出售，从第一本简装本被出售的时间直到最后一本小说被出售，简装本的出售量是精装本出售量的 9 倍。假如共有 441000 本书被出售，有多少本简装书被出售？

解答：由题意设简装本的出售量为 x 本，则依题意列出如下方程：

$$x + x/9 = 441000 - 36000$$

$$x = 364500$$

所以(C) is correct。

提示：对题目中的概念做概括或者替换，“Hard sold before = 36000, Hard = 9

paper, 若有 Hard sold before + Hard + paper = 441000。问 paper 等于多少？”。

例 8：Four cups of milk are to be poured into a 2-cup bottle and a 4-cup bottle. If each bottle is to be filled to the same fraction of its capacity, how many cups of milk should be poured into the 4-cup bottle?

- (A) 2/3
- (B) 7/3
- (C) 5/2
- (D) 8/3
- (E) 3

翻译：4 杯牛奶被倒入一个两杯的瓶中和一个 4 杯的瓶中。假如每个瓶中被充入的比例相同，有多少杯牛奶被倒入 4 杯瓶中？

解答：设倒入 4 杯瓶中的牛奶有 x 杯

$$\text{则 } \frac{x}{4} = \frac{4-x}{2}$$

$$x = 8/3$$

所以(D) is correct。

错误分析：注意“2-cup bottle”是可以装两杯的瓶，而“two bedroom”是有两个卧室的房间。

例 9：On a certain scale of intensity, each increment of 10 in magnitude represents a tenfold increase in intensity. On this scale, an intensity corresponding to a magnitude of 165 is how many times an intensity corresponding to a magnitude of 125?

- (A) 40
- (B) 100
- (C) 400
- (D) 1,000
- (E) 10,000

翻译：在某一强度等级中，每增加 10 级代

表强度增加 10 倍, 以这个标准, 对应于 165 级的强度是对应于 125 级的强度的多少倍?

解答: 本题在于阅读能力, magnitude 意为等级, 即 degree, tenfold 指 10 倍。

$$10^{\frac{165-125}{10}} = 10000$$

所以(E)is correct。

例 10: Of the positive integers that are multiples of 30 and are less than or equal to 360, what fraction are multiples of 12?

- (A) 1/6
(B) 1/5
(C) 1/3
(D) 2/5
(E) 1/2

翻译: 在 30 的倍数并且小于等于 360 的正整数中, 12 的倍数占了百分之几?

解答: 这些 30 的倍数并且小于等于 360 的数共 12 个。而即是 30 的倍数, 又是 12 的倍数, 它一定是 60 的倍数。在这 12 个正整数中, 60 的倍数占了百分之几呢? 60 的倍数在这 12 个数中共有 6 个, 因此 1/2 是正确答案。

所以(E)is correct。

提示: 关键在于理解“of”这个介词的作用。意为“在……中”, 而 that 引导的定语从句修饰这些正整数, fraction 意为分数, what fraction 译为百分之几? 读者应仔细体会这种问题方以及对句子结构的认识。

例 11: Town T has 20,000 residents, 60 percent of whom are female. What percent of the residents were born in Town T?

- (1) The number of female residents who were born in Town T is twice the

number of male residents who were not born in Town T.

- (2) The number of female residents who were not born in Town T is twice the number of female residents who were born in Town T.

翻译: 镇 T 有 2 万名居民, 60% 的人是女性, 出生于 T 镇居民的百分比是多少?

- (1) 出生于 T 镇的女性居民数目是不出生于 T 镇男性居民数目的 2 倍。
(2) 不出生于 T 镇的女性居民的数目是出生于 T 镇女性居民数目的 2 倍。

解答: 设出生于 T 镇的男性居民的百分比为 x , 出生于 T 镇的女性居民的百分比为 y , (1) 中仅说明 y 是 $40\% - x$ 的 2 倍, 无法求得 $x + y$; (2) 说明 $2y + y = 60\%$, $y = 20\%$, 但无法求得 x ; (1)+(2) 可以得到 x 的值, 所以可以得到 $x + y$ 的值。

所以(C)is correct。

提示: 解决本题的关键是对英语中倍数表达法的理解。以下是 GMAT 数学中常用的两种倍数表达法:

There are 2 times as many A as B.

A is 2 times as many as B.

它们分别表示什么呢? 考生经常搞不清楚。这里有一个方法来帮助记忆, 那就是第二个 as 后面的数永远是倍数后面的数。上面表达式中第二个 as 后面都是 B, 那么倍数 2 后面就是 B, 所以上面两种表达式都是 $A=2B$ 。

4.2 数学术语

一些比较生僻的数学术语是考生做题的巨大障碍, 例如 progression, binomial, denominator, complementary, angle 等, 若考生对这些词语的意思把握不到位, 会导致对整个题目的理解出现偏差。考生要在平时

尽可能地熟悉数学术语,做题时结合词根和上下文来猜测该数学术语的意思。

例 1: On a recent trip, Cindy drove her car 290 miles, rounded to the nearest 10 miles, and used 12 gallon of gasoline, rounded to the nearest gallon. The actual number of miles per gallon that Cindy's car got on this trip must have been between

- (A) 29/12.5 and 290/11.5
- (B) 295/12 and 185/11.5
- (C) 285/12 and 295/12
- (D) 285/12.5 and 295/11.5
- (E) 295/12.5 and 285/11.5

翻译: 在一次最近的旅行中, C 开车 290 英里, 四舍五入到 10 英里, 并且使用了 12 加仑的汽油, 四舍五入到加仑数。C 在此旅途中每加仑汽油所行驶的英里数一定在下列哪一个范围之内?

解答: to the nearest 指四舍五入, 则 C 开车的实际英里数在 285 至 295 之间, 而使用的加仑数在 11.5 至 12.5 加仑之间, 所以 C 每加仑汽油所行驶的英里数在下列范围内

295/11.5 to 285/12.5

所以(D)is correct。

提示: rounded to the nearest 指四舍五入, rounded to the nearest 10 miles 是指在个位数上四舍五入, 如 294.9 近似为 290, 而 295.1 近似为 300。

例 2: If a and b are positive integers. What is the value of $a + b$?

(1) $a/b=5/8$

(2) The greatest common divisor of a and b is 1.

翻译: 假如 a 和 b 都是正整数, $a + b$ 的值是多少?

(1) $a : b = 5 : 8$

(2) a 和 b 的最大公约数是 1。

解答: 由(1) $a:b=5:8$, $a + b$ 的值可能有多, 所以(1)单独不能回答这个问题; (2)由 a 和 b 的最大公约数是 1, 则 a 和 b 互质, 但(2)单独也无法回答这个问题。(1)+(2)可以得到 $a = 5, b = 8$, 则可以回答这个问题。

所以(C)is correct。

提示: common divisors 公约数

greatest common divisor 最大公约数

common multiple 公倍数

least common multiple 最小公倍数

例 3: Any decimal that has only a finite number of nonzero digits is a terminating decimal. For example, 24, 0.82, and 5.096 are three terminating decimals. If r and s are positive integers and the ratio r/s is expressed as a decimal, is r/s a terminating decimal?

(1) $90 < r < 100$

(2) $s = 4$

翻译: 仅有有限个非零数字的任何小数是有限小数, 例如, 24、0.82 和 5.096 是 3 个有限小数。假如 r 和 s 是正整数并且比率 r/s 被表示为一个小数, r/s 是有限小数吗?

解答: terminating decimal 有限小数

由(1) $90 < r < 100$ 无法说明 r/s 是否为有限小数; 由(2) $s = 4$, 已知 r 是正整数, 则无论 r 取什么值, $r/4$ 都是一个有限小数, 仅(2)就单独回答了问题。

所以(B)is correct。

提示: 注意 “No” 也是一种回答。

例 4: What is the least common denominator of $1/2$, $1/3$, and $1/4$?

- (A) 6
- (B) 8
- (C) 9
- (D) 12
- (E) 15

翻译: $1/2$ 、 $1/3$ 和 $1/4$ 最小公分母是多少?

解答: the least common denominator 最小公分母

所以 $1/2$ 、 $1/3$ 和 $1/4$ 的最小公分母为 12

所以(D)is correct。

例 5: In a certain formula, p is directly proportional to s and inversely proportional to r . If $p = 1$ when $r = 0.5$ and $s = 2$, what is the value of p in terms of r and s ?

- (A) s/r
- (B) $r/4s$
- (C) $s/4r$
- (D) r/s
- (E) $4r/s$

翻译: 在某一公式中, p 与 s 成正比, 并且与 r 成反比。假如当 $r = 0.5$ 且 $s = 2$ 时 $p = 1$, 下列哪一个是以 r 和 s 所表达的 p 的值?

解答: directly proportional to 正比
inversely proportional to 反比

设 $p = (s/r)a$

当 $r = 0.5$ 且 $s = 2$ 时 $p = 1$, 代入上式

$1 = (2/0.5)a$, $a = 1/4$, 所以 $p = s/4r$

所以(C)is correct。

提示: 本题的关键是理解:

directly proportional to 正比

inversely proportional to 反比

例 6: On a certain number line, if -7 is a distance 4 from n and 7 is a distance of 18 from n then $n = ?$

- (A) 25
- (B) 11
- (C) 3
- (D) -3
- (E) -11

翻译: 在某一数轴上, 假如 -7 与 n 的距离是 4, 并且 7 与 n 的距离是 18, 那么 $n = ?$

解答: 注意这句话的确切含义。

$$|-7 - n| = 4 \quad n = -11, n = -3$$

$$|7 - n| = 18 \quad n = -11, n = 25$$

所以 $n = -11$, (E)is correct。

提示: 注意比较下面两句话:

7 is a distance of 18 from n .

7 is the distance of 18 from n .

第一句用了不定冠词, 则该句应理解为

7 与 n 的距离是 18。第二句用了定冠词,

应理解为 7 是 18 和 n 之间的距离。

4.3 图表问题

有时 GMAT 考题中会出现图表题。由于我国考生在这一方面训练较少, 常常感到生疏。本节将比较详尽地对图表题给予讲

解。

常见的图表类型包括：

1. 表格

分类排列记录事项的文件。如：统计表，收支对照表等。如表 4.1 所示：

表 4.1

科目 学号	语文	数学	英语	生物
#01	87	98	96	80
#02	78	89	87	93
#03	91	99	94	90

2. 扇形图

表示部分与整体的关系，通常以百分比表示图中的每个部分。图中整个圆代表 100%，占总数一定百分比的一个量以相同的比例用一个“扇面”表示，扇面越大，所占的比率就越高。如图 4.2 所示：

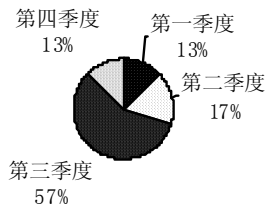


图 4.1

3. 柱形图

主要利用柱子的长度或者高度来进行比较，每个柱子可能表示一个量，也可能将其分割成几段以表示不同的量。如图 4.3 所示：

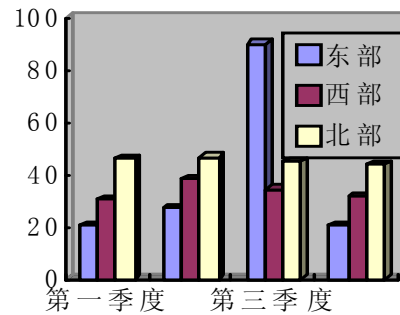


图 4.2

4. 线形图

主要用来描述某一量的连续变化过程，通常以时间做为变化参数。有些图中可能不止一条曲线，这些曲线在变化时可能会交叉或者重叠，且交叉重叠部分又往往是 ETS 的考点。如图 4.4 所示。

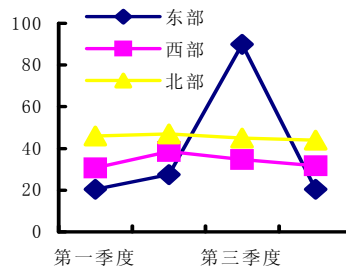


图 4.3

解答图表时考生应该注意：

1. 首先对整个图表做一大致了解，但不要深入到图表中。一开始就希望对其全面了解，会浪费大量的时间，且获得的许多信息又是无用的。考生要做到“按需索取”。

2. 注意图表中的单位。

3. 不要搞混小数和百分数，例如：0.6%实际上等于 0.006；不要搞混百分数与原始数，例如：把 30%当作 30 来处理。

例 1: The table below shows the number of

tickets sold during each of the first 4 weeks after a movie was released. The producer of the movie received 10 percent of the revenue from every ticket sold with a guaranteed minimum of \$200,000 per week for the first 4 weeks. If tickets sold for \$4 each, how much did the producer receive for the first 4 weeks?

表 4.2

Week	Number of Tickers Sold
1	1,000,000
2	1,000,000
3	750,000
4	250,000

- (A) \$800,000 (B) \$900,000
 (C) \$1,000,000 (D) \$1,200,000
 (E) \$1,300,000

翻译: 图 4.5 所展示的是在一个电影公映后的前 4 周中每一周销售的电影票数。电影的出品人获得每张票销售收入的 10%，并且在前 4 周的每一周有一个被保证的最低收入，为 200000 美元。假如票以每张 4 美元出售，电影出品人在前 4 周得到了多少钱？

解答: 本题意为出品人获得售票收入的 10% 当最低收入超过 200000 美元时，若售票收入的 10% 不足 200000 美元则出品人也获得保证的 200000 美元。

由上表观察可得前 3 周售票收入的 10% 均超过 200000 美元，而第四周售票收入的 10% 为：

$$250000 \times 4 \times 10\% = 100000$$

不足 200000 美元，则出品人第四周获得 200000 美元，所以出品人前 4 周共获得：

$$(1000000 + 1000000 + 750000) \times 4 \times 10\%$$

$$+ 200000$$

$$= 1300000 \text{ 美元}$$

所以(E) is correct.

提示: 注意票房收入不同时，出品人收入有两种计算方法。

例 2: The table below shows the average (arithmetic mean) price per dozen of the large grade A eggs sold in a certain store during three successive months. If $\frac{2}{3}$ as many dozen were sold in April as in May, and twice as many were sold in June as in April, what was the average price per dozen of the eggs sold over the three-month period?

表 4.3

Month	Average Price per Dozen
April	\$1.26
May	\$1.20
June	\$1.08

- (A) \$1.08 (B) \$1.10
 (C) \$1.14 (D) \$1.16
 (E) \$1.18

翻译: 表 4.6 所展示的是某一商店连续 3 个月的 A 级鸡蛋每打的平均销售价格。假如 4 月销售的鸡蛋的打数是 5 月的 $\frac{2}{3}$ ，并且 6 月的销售的鸡蛋的打数是 4 月的两倍，3 个月期间每打鸡蛋出售的平均价格是多少？

解答: 设 4 月的销售打数为 x ，则 6 月的销售打数为 $2x$ ，5 月销售的打数为 $3x/2$ ，则 3 个月中每打鸡蛋的平均销售价格为：

$$(1.26x + 1.2 \times 3x/2 + 1.08 \times 2x) / (x + 3x/2 + 2x) = 1.16$$

所以(D) is correct.

4.4 投资问题

1. 折扣(Discount)

商品按原定价格扣除百分之几出售。If a price is discounted by n percent, the price becomes $(100-n)$ percent of the original price.

2. 利息(Interest)

借款人支付给贷款人的报酬。利息可分为单利(simple interest)和复利(compound interest)两种计算方法。

3. 单利(Simple Interest)

计算利息的一种方法。不管期限长短, 仅按本金(principal)计算利息, 其所生利息不再加入本金重复计算利息。

单利利息=本金 $p \times$ 利率 $r \times$ 时间 t

单利本金利息和 $A=p(1+rt)$

4. 复利(Compound Interest)

单利的对称。经过一定的期限, 将所生利息加入本金再计利息, 逐期滚算, 俗称“利滚利”。

复利本金利息和 $=p(1+r)^t$

复利利息 $A=p[(1+r)^t-1]$

5. 利率(Rate or Percent of Interest)

亦称“利息率”, 指一定时期内利息额同贷出金额的比率, 有年利率、月利率和日利率。

4. 利润(Profit)

利润=收入-消耗

利润=销售收入-进价

提示: 单利与复利计算时, 一定要注意单位换算, 如是以半年为单位计算复利, 还是以三个月为单位计算复利。

例 1: A family made a down payment of \$75 and borrowed the balance on a set of

encyclopedias that cost \$400. The balance with interest was paid in 23 monthly payments of \$16 each and a final payment of \$9. The amount of interest paid was what percent of the amount borrowed?

- (A) 6%
- (B) 12%
- (C) 14%
- (D) 16%
- (E) 20%

翻译: 一个家庭分期付款买了一套价值 400 美元的百科全书, 首期付款为 75 美元, 余款和利息在 23 个月中每个月付 16 美元并且最后的付款为 9 美元, 所付的利息的量是借款量的百分之多少?

解答: 除去首期付款, 该家庭共付了:

$$23 \times 16 + 9 = 377 \text{ 美元}$$

$$\text{则利息为 } 377 - (400 - 75) = 52 \text{ 美元}$$

$$\text{利息是借款量的 } 52/325 = 16\%$$

所以(D) is correct.

提示:

$$\text{利息率} = \frac{\text{利息}}{\text{贷款数}}$$

$$\text{贷款数} = \text{应付价格} - \text{首付额}$$

例 2: Last year Luis invested x dollars for one year, half at 8 percent simple annual interest and the other half at 12 percent simple annual interest. Now he wants to reinvest the x dollars for one year in the same two types of investments, but the lower rate has decreased. If the higher rate is unchanged, what fraction of the x dollars must be reinvest at the 12 percent rate so that the total interest earned from the x dollars will be the same for both years?

- (1) The lower rate is now 6 percent.
 (2) The total amount of interest earned from the two investments last year was \$3,000.

翻译: 去年 L 投资 x 美元, 时间为 1 年, 一半以 8% 的年单利, 另一半以 12% 的年单利。现在他要重新投资 x 美元, 时间也为一年并且以上面的两种相同类型的投资方式, 但是较低的利率已经下降了。假如较高的利率不变, x 美元中的多大比例必须以 12% 的利率投资以致从 x 美元中所挣得的所有利息对两年而言是相同的?

- (1) 较低的利率现在是 6%。
 (2) 去年从两种投资中挣得的总利息为 3000 美元。

解答: 由(1)中得知较低利率为 6%, 设以 12% 投资的百分比为 m , 则得到如下等式

$$\begin{aligned} x/2(8\% + 12\%) \\ = mx \times 12\% + (x - mx) \times 6\% \end{aligned}$$

可以解得 m 的值, 因为 x 可以消去, 所以(1)单独回答了上面的问题; 由(2)无法得到以 12% 投资的百分比。

所以(A) is correct。

提示: 题目中给出的是未知数 x 和比例, 而条件(2)用的是一个具体数字, 因此条件(2)应该排除。

例 3: A 2-year certificate of deposit is purchased for k dollars. If the certificate earns interest at an annual rate of 6 percent compounded quarterly, which of the following represents the value, in dollars, of the certificate at the end of the 2 years?

- (A) $(1.06)^2 k$

- (B) $(1.06)^8 k$
 (C) $(1.015)^2 k$
 (D) $(1.015)^8 k$
 (E) $(1.03)^4 k$

翻译: 假如一个两年的定期存款以 k 美元被购买。假如那个定期存款以年利率 6%, 以季度复利得到利息, 下列哪一个是这个定期存款在两年末以美元为单位所代表的价值?

解答: 很多考生难以理解本题第一句话: 为什么定期存款以 k 美元被购买, 其实 certificate of deposit 简称 CD, 是一种美国金融市场上可流通的一种金融产品, 以 k 美元购买, 即指这个定期存单上有 k 美元, 因为以季度复利, 所以每季度利息为:

$$6\% \div 4 = 1.5\%$$

而两年有 8 个季度, 所以两年末这张 CD 的值为:

$$k(1.015)^8$$

所以(D) is correct。

提示: 注意 6% 是年利率, 而季度复利得到利息, 因此季度利息为 1.5%, 单利与复利计算时, 一定要注意时间单位的换算。

例 4: Mary invested \$ 8,400 for 6 months in a certificate of deposit paying 9.25 percent simple annual interest. How much more interest would Mary have received if the interest rate on this certificate had been 9.75 percent simple annual interest?

- (A) \$2.10
 (B) \$21.00
 (C) \$42.00
 (D) \$210.00
 (E) \$420.00

翻译: M 把 8400 美元投资入一个利息为

9.25%的年利率的定期存款，存期为6个月。假如这个定期存单的年利率为9.75%，那么M可以多获得多少利息？

解答： $8400 \times 9.75 \times 0.5 - 8400 \times 9.25 \times 0.5$
 $= 4200 \times (9.75\% - 9.25\%)$
 $= 21$ 美元

所以(B) is correct.

错误分析：9.75%和9.25%是年利率，而存款期是半年。

例 5：On July 1, 1982, Ms. Fox deposited \$10,000 in a new account at the annual interest rate of 12 percent compounded monthly. If no additional deposits or withdrawals were made and if interest was credited on the last day of each month, what was the amount of money in the account on September 1, 1982?

- (A) \$10,200
- (B) \$10,201
- (C) \$11,100
- (D) \$12,100
- (E) \$12,544

翻译：在1982年7月1日，F女士在一个新账户上存了10000美元以年利率为12%且以月为单位计算复利。假如她既没有存钱也没有取钱并且利息在每个月最后一天加入账户，在1982年9月1日账户中的钱有多少？

解答：年利率为12%，则月利为1%
 两个月后账户中的钱有：
 $10000(1+1\%)^2 = 10201$ 美元
 所以(B) is correct.

4.5 本章习题精选

1. Is $\triangle MNP$ isosceles?

- (1) Exactly two of the angles, $\angle M$ and $\angle N$, have the same measure

(2) $\angle N$ and $\angle P$ do not have the same measure.

2. A coal company can choose to transport coal to one of its customers by railroad or by truck. If the railroad charges by the mile and the trucking company charges by the ton, which means of transporting the coal would cost less than the other?

- (1) The railroad charges \$5,000 plus \$0.01 per mile per railroad car used, and the trucking company charges \$3,000 plus \$85 per ton.
- (2) The customer to whom the coal is to be sent is 195 miles away from the coal company.

3. The table above gives the number of calories and grams of protein per kilogram of foods S and T. If a total of 7 kilograms of S and T are combined to make a certain food mixture, how many kilograms of food S are in the mixture?

- (1) The mixture has a total of 12,000 calories.
- (2) The mixture has a total of 810 grams of protein.

表 4.4

Food	Number of Calories per Kilogram	Number of Grams of Protein per Kilogram
S	2,000	150
T	1,500	90

4. In Country S, if 60 percent of the women aged 18 and over are in the labor force, how many million women are in the labor force?

- (1) In Country S, women comprise 45

- percent of the labor force.
- (2) In Country S, there are no women under 18 years of age in the labor force.
5. In order to give his customers a 25 per cent discount on the price and still net a 25 per cent profit on the cost of an item, at what price should a merchant mark an item if it cost him \$16.80?
- (A) \$21.00
(B) \$21.90
(C) \$25.20
(D) \$26.25
(E) \$28.00
6. A warehouse had a square floor with area 10,000 square meters. A rectangular addition was built along one entire side of the warehouse that increased the floor area by one-half as much as the original floor area. How many meters did the addition extend beyond the original building?
- (A) 10
(B) 20
(C) 50
(D) 200
(E) 500
7. A digital wristwatch was set accurately at 8:30 a.m. and then lost 2 seconds every 5 minutes. What time was indicated on the watch at 6:30 p.m. of the same day if the watch operated continuously until that time?
- (A) 5:56
(B) 5:58
(C) 6:00
(D) 6:23
(E) 6:26
8. If $xyz \neq 0$, and x percent of y percent of z is t , then $z = ?$
- (A) $\frac{100t}{xy}$
(B) $\frac{1,000t}{xy}$
(C) $\frac{10,000t}{xy}$
(D) $\frac{xy}{10,000t}$
(E) $\frac{10,000xy}{t}$
9. A merchant sells an item at a 20 percent discount, but still makes a gross profit of 20 percent of the cost. What percent of the cost would the gross profit on the item have been if it had been sold without the discount?
- (A) 20%
(B) 40%
(C) 50%
(D) 60%
(E) 75%
10. A decorator bought a bolt of defective cloth that she judged to be $\frac{3}{4}$ usable, in which case the cost would be \$0.80 per usable yard. If it was later found that only $\frac{2}{3}$ of the bolt could be used, what was the actual cost per usable yard?
- (A) \$0.60
(B) \$0.90
(C) \$1.00
(D) \$1.20

- (E) \$1.70
11. Sue is now 10 years younger than Jane. If in 5 years, Jane will be twice as old as Sue, how old will Sue be in 3 years?
- (A) 6
(B) 8
(C) 11
(D) 14
(E) 18
12. The result obtained when x is multiplied by y is equal to ten times the result obtained when y is subtracted from x . If y equals 5, what does x equal?
- (A) 50
(B) 25
(C) 15
(D) 10
(E) 5
13. Laura borrowed \$240, interest free, from her parents to pay for her college education. If she pays back $2\frac{1}{2}$ percent of this amount quarterly, and has already paid \$42.00, for how many months has she been paying back her loan?
- (A) 6
(B) 7
(C) 19
(D) 21
(E) 24
14. In a basketball game, Tim scored 10 points more than Joe, but only half as many points as Ken. If the three players scored a combined total of 50 points. How many points did Tim score?
- (A) 10
(B) 15
(C) 16
(D) 24
(E) 25
15. Mary invested \$8,400 for 6 months in a certificate of deposit paying $9\frac{1}{4}$ percent simple annual interest. How much more interest would Mary have received if the interest rate on this certificate had been $9\frac{3}{4}$ percent simple annual interest?
- (A) \$2.10
(B) \$21.00
(C) \$42.00
(D) \$210.00
(E) \$420.00
16. A magazine subscription is offered for 1 year at \$12 or for 3 years at \$29. If the rates do not change during the 3 years, the cost of a 3-year subscription is approximately what percent less than the total cost of 3 annual subscriptions?
- (A) 59%
(B) 58%
(C) 32%
(D) 19%
(E) 16%
17. On a legislative committee, the number of males is 2 fewer than twice the number of females. If one male was replaced by a female, there would be an equal number of

males and females on the committee. How many members are on the committee?

- (A) 4
- (B) 8
- (C) 10
- (D) 13
- (E) 18

18. The table below shows the cancellation fee schedule that a travel agency uses to determine the fee charged to a tourist who cancels a trip prior to departure. If a tourist canceled a trip with a package price of \$ 1,700 and a departure date of September 4, on what day was the trip canceled?

表 4.5

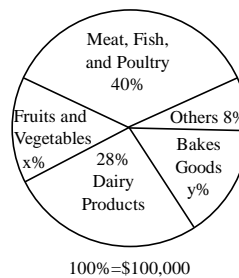
CANCELLATION FEES	
Days Prior to Departure	Percent of Package Price
46 or more	10%
45-31	35%
30-16	50%
15-5	65%
4 or fewer	100%

- (1) The cancellation fee was \$ 595.
- (2) If the trip had been canceled one day later, the cancellation fee would have been \$ 255 more.

19. According to the graph below, the sale of fruits and vegetables in Store S last week accounted for what percent of the total sales income for the week?

- (1) Last week the total income from the sale of fruits and vegetables in Store S was \$16,000.
- (2) $x = 2y$

DISTRIBUTION OF SALES INCOME FOR STORE SLAST WEEK



100%=\$100,000

图 4.4

参考答案:

- 1~5 AEDEE
- 6~10 CECCB
- 11~15 BDDBB
- 16~19 DCCD

第5章 马虎问题

马虎问题是大多数我国考生在考试中不能取得优异成绩的很大原因,也是备考者应该充分重视的一个问题。

本章试图将广大考生常见的马虎问题进行分类,附以详细讲解的例题以及习题,以引起广大考生的注意,减少因马虎而造成的不必要的失分。

5.1 单位的换算

注意一些题所给的数量单位与最后所问的数量的单位不同,一定要在答题时将其进行转换。

常见的转换包括货币的转换(如 \$ 1 = 100 cents)、时间的转换(如 1 hour = 60 minutes = 3600 seconds)以及距离的转换(如 1 kilometer = 1000 meters)。

一般情况下,单位的转换出现在 Problem Solving 题中,但有时也出现在 Data Sufficiency 题中。对于 Problem Solving 题,如果题目中出现不同单位,一般会在答案中体现,如对时间的转换(分子或分母会有 60,因为时间的进制是 60)等。

不同单位的转换也可以作为排除法的一个依据。例如只有相同单位才可以相互加减,所以 m 小时加上 n 分钟是 $m + n/60$ 小时,而不是 $m + n$ 小时。

例 1: A machine costs m dollars per day to maintain and n cents for each unit it produces. If the machine is operated 7 days a week and produces r units in a week, which of the following is the total cost, in dollars, of operating the machine for a week?

- (A) $7m + 100nr$
- (B) $(700m + nr)/100$
- (C) $7m + nr$
- (D) $(7m + 100nr)/100$
- (E) $700mnr \left(\frac{\pi}{2} - \theta \right)$

翻译: 某机器每天需要 m 美元的成本来维持,并且每生产一件产品需要 n 美分的成本。假如该机器一周 7 天工作并且每周生产 r 件产品,下列哪一项是以美元为单位操作该机器一周的总成本?

解答: 总成本应由如下算式得出:

$$(700m + nr)/100$$

所以(B) is correct。

错误分析: 本题涉及到美元和美分的单位转换。很多粗心的考生忽略了这一点,不假思索地选择了 C 选项。有的考生注意到了美元与美分的区别,却错误地选择了 A 选项。

例 2: A certain car increased its average speed by 5 miles per hour in each successive 5-minute after the first interval. If in the first 5-minute interval its average speed was 20 miles per hour, how many miles did the car travel in the third 5-minute interval?

- (A) 1.0
- (B) 1.5
- (C) 2.0
- (D) 2.5
- (E) 150

翻译: 一辆小汽车在第一个 5 分钟后的每 5 分钟内增加其平均速度 5 英里/小时。假如在第一个 5 分钟内的平均速度为每小时 20 英里。在第三个 5 分钟内, 小汽车行驶了多少英里?

解答: 第三个 5 分钟小汽车的速度为:

$$20 + 5 + 5 = 30 \text{ 英里/小时}$$

第三个 5 分钟行驶的里程为:

$$30 \times 5 \times (1/60) = 2.5 \text{ 英里}$$

所以(D) is correct.

错误分析: 有些考生求出了第三个 5 分钟小汽车的速度, 却忽略了小时与分钟的单位转换, 因此前功尽弃。

5.2 比例问题

常见的比例问题有两种形式:

1. 二分型比例问题。题目中涉及的概念往往一分为二, 例如产品有正品和次品之分, 驾驶者有正常驾驶和超速者之分。问题的难点在于这个概念有时又有新的分类, 而且新的分类与旧的分类常常相互交叉。例如产品有售出的和未售出的, 售出的产品中可能有正品也可能有次品, 同样未售出的产品中可能有正品也可能有次品。

题目中的概念可以画成表格, 将已知的信息填入表格, 这样整个题目就可以变得清晰明朗。

2. 百分比的变化。GMAT 数学考试中, 另外一类常见的题型是考查从一个量到另外一个量的百分比变化。该类问题最关键的是搞清分母是谁。

例 1: In each production lot for a certain toy, 25 percent of the toys are red and 75 percent of the toys are blue. Half the toys are size A and half are size B. If 10 out of a lot of 100 toys are red and size A, how many of the toys are blue and size B?

- (A) 15
- (B) 25
- (C) 30
- (D) 35
- (E) 40

翻译: 在某一玩具的每个产品生产地, 25% 的玩具是红色, 75% 的玩具是蓝色。50% 的玩具是 A 型的, 50% 的玩具是 B 型的。假如一批 100 个玩具中的 10 个是红色并且是 A 型, 那么多少玩具是蓝色并且是 B 型?

解答: 100 个玩具中有 10 个是红色并且是 A 型, 则有 15 是红色、B 型, 而 B 型共 50 个, 所以蓝色 B 型有 35 个。

所以(D) is correct.

错误分析：本题将玩具两次分类，而且两次分类相互交叉，你中有我，我中有你，让很多考生很糊涂。

表 5.1

	Size A	Size B
Blue	$100/2-10=40$	$75-40=35$
Red	10	$25-10=15$

如果能将题目中的概念可以画成表格，整个题目将变得非常简单。

例 2： On a certain road, 10 percent of the motorists exceed the posted speed limit and receive speeding tickets, but 20 percent of the motorists who exceed the posted speed limit do not receive speeding tickets. What percent of the motorists on that road exceed the posted speed limit?

- (A) $10\frac{1}{2}\%$
 (B) 22%
 (C) $12\frac{1}{2}\%$
 (D) 30%
 (E) 15%

翻译：在某一条路上，10%的汽车驾驶者超过标准的速度限制后得到一张超速罚单，但是20%的超速驾驶者未接到超速罚单，那条路上超速驾驶者的百分比是多少？

解答：由超速的人中未接到超速罚单的人有20%可得，超速的人接到罚单的比率为80%，所以超速者所占百分比为：

$$10\%/80\%=12.5\%$$

所以(C) is correct.

错误分析：本题中出现两个百分数，让很多考生摸不着头脑。对于比例问题，关键是搞清分母是谁。就本题而言，10%是

超速度限制并且得到罚单的驾驶者占所有驾驶者的比例，20%是超速但未接到超速罚单的驾驶者占超速者的比例。因此，速的人中有80%接到罚单，他们占所有驾驶者的10%。

提示：比例问题关键是搞清分母是谁。考生可以借助表格和线段来表达。

例 3： The market value of a certain machine decreased by 30 percent of its purchase price each year. If the machine was purchased in 1982 for its market value of \$8,000, what was its market value two years later?

- (A) \$8,000
 (B) \$5,600
 (C) \$3,200
 (D) \$2,400
 (E) \$8,00

翻译：某一机器的市场价值每年下降其购买价格的30%，假如该机器1982年购买时的市场价值为8000美元，两年后其市场价值是多少？

解答： $8000 - 2 \times 8000 \times 30\% = 3200$ 美元

所以(C) is correct.

错误分析：本题第一句话说明每年下降的量是相同的，所以不能通过 $800 \times (1 - 30\%)^2$ 求得答案。

例 4： A merchant purchased a jacket for \$ 60 and then determined a selling price that equaled the purchase price of the jacket plus a markup that was 25 percent of the selling price. During a sale, the merchant discounted the selling price by 20 percent and sold the jacket. What was the merchant's gross profit on this sale?

- (A) \$0
 (B) \$3
 (C) \$4
 (D) \$12
 (E) \$15

翻译: 一个商人以 60 美元购买了一件夹克, 并且决定销售价格为购买价格加上一个等于销售价格 25% 的涨价。在销售期间, 商人对销售价格打了 20% 的折扣并且售出了这件夹克, 这次销售中商人的总利润是多少?

解答: markup n. “涨价”的意思

设销售价格(selling price)为 x 美元, 该价格为购买价格加上销售价格的 25%, 则可以得到如下方程:

$x=60+x \times 25\%$, $x=80$ 美元, 则总利润为 $80 \times (1-20\%) - 60 = 4$ 美元

所以(C) is correct。

错误分析: 注意 25% 是销售价格的 25%, 而不是 \$60 的 25%。

例 5: Mrs. Jones sold two houses for \$96,000 each. One house was sold at a 20 percent loss and the other at a 20 percent gain. How many dollars did Mrs. Jones gain in total?

- (A) 0
 (B) 8,000
 (C) -8,000
 (D) 12,000
 (E) -12,000

翻译: 琼斯太太以每所 \$96,000 的价格卖了 two 所房屋。一座房子以 20% 的亏损售出, 另一座房子以 20% 的获利售出。

解答: 设获利房屋价为 x , 则:

$$x(1+20\%)=96,000$$

设亏损房屋价为 y , 则:

$$y(1-20\%)=96,000$$

$$\text{则 } x = \frac{96,000}{(1+20\%)} = 80,000$$

$$y = \frac{96,000}{(1-20\%)} = 120,000$$

因此, Mrs. Jones 获利为:

$$96000 \times 2 - 80000 - 120000 = -8000$$

所以(C) is correct。

错误分析: 注意本题不同于: Mrs. Jones **bought** two houses for \$96,000 each. One house was sold at a 20 percent loss and the other at a 20 percent gain. How many dollars did Mrs. Jones gain in total?

例 6: Ms. Smith got an 8 percent cost of living raise of \$ 20 per week. What is Ms. Smith's new weekly salary?

- (A) 270
 (B) 250
 (C) 220
 (D) 200
 (E) 180

翻译: Ms. Smith 获得每周 20 美元的相当于 8% 生活费用的工资增加。Ms. Smith 的新工资是多少?

解答: cost of living 是“生活费用”的意思, 而在此即指 salary 这 8% cost of living raise, 即 \$20。 $20 \div 8\% = 250$, new weekly salary 为: $250+20=270$
 所以(A) is correct。

5.3 统计元素的个数

在集合、数列和概率问题中, 经常需要统计元素的个数。在统计时, 经常会犯这种错误: 没有包括边界值或者边界值多数了一次。

例如等差数列 $\{1, 3, 5, 7, \dots, 999\}$ 中元

素的个数是:

$$N = (999 - 1) / 2 + 1 = 500$$

例如 $-4 < M < 4$, M 是整数, 问 M 有多少可能性? M 可能是 -3 、 -2 、 -1 、 0 、 1 、 2 、 3 中的一个。一个 M 有 7 种可能性。

例 1: Raffle tickets numbered consecutively from 101 through 350 are placed in a box. What is the probability that a ticket selected at random will have a number with a hundreds digit of 2?

- (A) $\frac{2}{5}$
 (B) $\frac{2}{7}$
 (C) $\frac{33}{83}$
 (D) $\frac{99}{250}$
 (E) $\frac{100}{249}$

翻译: 彩票由 101 至 350 被放置在一个盒子中, 从盒子中随机选择一张彩票, 百位数字是 2 的概率是多少?

解答: hundreds digit 百分数, units digit 指个位数, tenths digit 指十位数。在 101 至 350 中, 百位数字是 2 的一共有 100 个, 而 101 至 350 中共有 250 个数字, 百分数字是 2 的概率为:

$$P = \frac{100}{250} = \frac{2}{5}$$

所以(A) is correct.

错误分析: 注意 101 至 350 中共 250 个数字, 而不是 $350 - 101 = 249$ 个数字。

例 2: City Y has installed 30 parking meters at 15-foot intervals along a straight street. What is the number of feet between the first meter and the last meter?

- (A) 200
 (B) 420
 (C) 435
 (D) 450
 (E) 465

翻译: 城市 Y 沿一条直的街道每隔 15 英尺安装了 30 个停车计价器, 第一个计价器与最后一个计价器之间的英尺数是多少?

解答: 在第一个计价器与最后一个计价器间隔了 29 个 15 英尺的距离。读者体会此类考题, 关键在于 $(n+1)$ 个元素才能隔开 n 个距离, 例如 3 个 meter 才能分开 2 个 15 英尺的距离。

$$29 \times 15 = 435$$

所以(C) is correct.

错误分析: 有些考生简单地认为在第一个计价器与最后一个计价器间隔了 30 个 15 英尺的距离。

5.4 考虑不全面

本节考虑的马虎问题包括:

(1) 两种和两种以上的可能性, 但只考虑了其中一种。

(2) Data Sufficiency 题中, 条件 1 能够充分回答问题, 即选择了 A 选项, 根本没有考虑条件 2。

(3) 运动问题中考虑不完全, 例如只考虑 A 物体运动, 没有考虑到 B 物体同时也在运动。

对于运动问题, 考生可以绘制一个整个过程的草图帮助理解, 以减少错误的可能性。

例 1: A, B, and C are points on a line. The distance between A and B is twice the distance between A and C. The distance between C and B is 10. What is the distance between A and B?

- (A) 5
 (B) $\frac{20}{3}$
 (C) 20
 (D) $\frac{20}{3}$ or 20
 (E) 5 or 20

翻译: A、B、C 是一条直线上的点，A 与 B 之间的距离是 A 与 C 之间距离的 2 倍，C 与 B 的距离是 10。

解答: 本题存在两种情况，如图 5.1 所示：

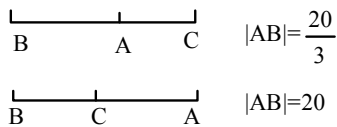


图 5.1

所以(D) is correct.

错误分析: 很多考生只是考虑到两种情况中的一种，错误地选择了选项 B 或 C。也有一些考生虽然考虑到有两种情况，却懒得动手作图，因此错误地选择了答案 E。

提示: 对于数轴问题和距离问题，最简单也是最安全的方法是数形结合，然后根据图进行计算。

例 2: Each number in the arrangement below is obtained from the two nearest numbers in the column immediately to the left by subtracting the upper number from the lower number. What is the value of z ?

- (1) $x=7$
 (2) $t=5$

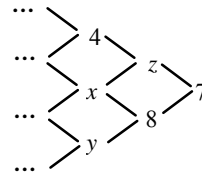


图 5.2

翻译: 如图 5.2 所排列的每一个数字都由左边一栏中两个最近的数字得到，通过较低的数字减去较高的数字， z 的值是多少？

解答: 由上面的定义可得， $z=x-4$ 或者 $8-z=t$ ，
 (1)由已知 $x=7$ ，则 $z=3$ ，可以回答上面的问题；
 (2)由已知 $t=5$ ，则 $z=8-t=3$ ，也可以回答上面的问题。

所以(D) is correct.

错误分析: 许多考生发现数据 1 足以答题，即仓促地选择 A，而不再管数据 2。

提示: 考虑两个数据中的一个(一般是数据 1)，而不管另外一个，是 Data Sufficiency 题的大忌。

例 3: What is the value of the two-digit number x ?

- (1) The sum of the two digits is 4.
 (2) The difference between the two digits is 2.

翻译: 两位数 x 的值是多少？

解答: 单独由(1)或(2)均无法回答上面的问题；由(1)+(2)可以得到两个方程，设十位数和个位数分别为 m 和 n ，则：

$$m + n = 4$$

$$m - n = 2$$

$m = 3, n = 1$ ，但可以组成两个十位数即 13 和 31，(1)和(2)也无法回答上面问题。

所以(E) is correct.

错误分析: 有的考生计算出 $m = 3, n = 1$ ，即认为该两位数是 31，而忽略了 13，

从而错误地选择了答案 C, 甚为可惜。

例 4: A certain ball was dropped from a window 8 meters above a sidewalk. On each bounce it rose straight up exactly one-half the distance of the previous fall. After the third bounce the ball was caught when it reached a height of exactly 1 meter above the sidewalk. How many meters did the ball travel in all?

- (A) 21
(B) 19
(C) 17
(D) 15
(E) 13

翻译: 一个球从人行道上 8 米高的窗户处落下。每次反弹都是上一次下落距离的 $\frac{1}{2}$, 在第三次反弹后, 球在高于人行道的 1 米处被抓住, 该球共走了多少米?

解答: 从窗户上落地共 8 米, 而第一次反弹高度为 4 米, 来回共 8 米; 第二次反弹高度为 2 米, 来回共 4 米; 第三次反弹高度为 1 米, 达到第三次反弹的最高点时被抓住, 所以该球共走了:

$$8+4+2+2+2+1=21 \text{ 米}$$

所以(A)is correct。

错误分析: 有的考生缺乏对整个过程的全面考虑, 只考虑到下落过程而忽略了球反弹的过程, 或者相反。因此, 计算该球共走了:

$$8+4+2+1=15 \text{ 米}$$

例 5: A boy walking along a road at 3 kilometers per hour is overtaken by a truck traveling at 40 kilometers per hour. If the truck breaks down 1 kilometer

beyond where it passes the boy, how many minutes after the breakdown does the boy reach the truck?

- (A) $21\frac{1}{2}$
(B) 20
(C) $18\frac{34}{37}$
(D) $18\frac{26}{43}$
(E) $18\frac{1}{2}$

翻译: 一个以每小时 3 公里的速度行走的男孩被一辆以 40 公里/小时行驶的卡车超过。假如卡车在通过该男孩的一公里远处时出了故障, 在出故障后的多少分钟该男孩到达此卡车?

解答: 设该男孩在卡车出了故障后 x 分钟到达卡车, 则可以列出以下方程:

$$3 \times (x/60 + 1/40) = 1$$

$$x = 18.5 \text{ 分钟}$$

所以(E)is correct。

错误分析: 有的考生没有对整个过程做全面的考虑, 即认为本题等同于“一个以每小时 3 公里的速度行走的男孩用多少分钟可以完成一公里的距离”, 因此列出以下方程:

$$3 \times (x/60) = 1$$

$$x = 20 \text{ 分钟}$$

如果考生能绘制一个描述整个过程的草图, 便可以轻易地发现错误。

提示: 对于运动问题, 图形或者线段是最安全和最直观的方法。建议考生在每个运动问题中都使用。

5.5 答非所问

有的考生在没有读完题或者没有完全理解问题的情况下, 即开始计算。尽管思路

可能是正确的，但由于考生答非 ETS 所问，能做对的可能性将非常小。

本节选择的几个典型例题，都是考生非常容易答错的问题。希望大家能够减少此类错误。

例 1: Diana bought a stereo for \$530, which was the retail price plus a 6 percent sales tax. How much money could she have saved if she had bought the stereo at same retail price in a neighboring state where she would have paid a sales tax of 5 percent?

- (A) \$1.00
- (B) \$2.65
- (C) \$4.30
- (D) \$5.00
- (E) \$5.30

翻译: 黛安娜花\$530 购买了一个音响，为零售价加了上 6%的购物税。假如在购物税是 5%的邻近州以相同的零售价购买，她可以省多少钱？

解答: 设音响零售价为 x ，则

$$x(1 + 6\%) = 530, x = 500$$

则节省的钱为：

$$500 \times (6\% - 5\%) = 5$$

所以(D)is correct.

错误分析: 本题要求的是“可以省多少钱”，而不是“假如购物税是 5%，需要付多少税”。如果清楚了这一点，本题应该没有多少难度。

提示: 有的考生还没有看完考题，即把考题与自己曾经见过的某道考题划上等号，轻易地做出回答。这实在是 GMAT 数学考试大忌中的大忌。

例 2: If $1/2$ of the air in a tank is removed with

each stroke of a vacuum pump, what fraction of the original amount of air has been removed after 4 strokes?

- (A) $15/16$
- (B) $7/8$
- (C) $1/4$
- (D) $1/8$
- (E) $1/16$

翻译: 假如真空阀每次抽出某一容器中的 $1/2$ 的空气，在抽 4 次后多大比例的空气被抽出？

解答: 由于每次抽出容器中空气的 $1/2$ ，则 4 次分别抽出了 $1/2$ ， $1/4$ ， $1/8$ ， $1/16$ 的空气，所以：

$$1/2 + 1/4 + 1/8 + 1/16 = 15/16$$

所以(A)is correct.

错误分析: 本题并没有多少难度。关键在于要求的是“多大比例的空气被抽出”，而不是“剩余多少空气”。然而，许多考生选择了答案 E。

提示: 答非所问，要对也难。

例 3: A certain basketball team that has played $2/3$ of its games has a record of 17 wins and 3 losses. What is the greatest number of the remaining games that the team can lose and still win at least $3/4$ of all of games?

- (A) 3
- (B) 6
- (C) 5
- (D) 4
- (E) 3

翻译: 某一篮球队已经完成了其 $2/3$ 的比赛，成绩是 17 胜 3 负。该队最多还可以输多少场比赛但仍然至少赢 $3/4$ 的比赛？

解答: $2/3$ 的比赛场数是 20 场，所以共 30

场比赛, 至少赢 $3/4$ 的比赛则至少赢 $30 \times (3/4) = 23$ 场, 所以该队最多可以输 $30 - 23 - 3 = 4$ 场比赛。

所以(D) is correct.

错误分析: 本题的关键是 “the remaining games”。许多考生计算出共 30 场比赛, 也计算出至少赢 $30 \times (3/4) = 23$ 场, 便粗心地认为可以输 $30 - 23 = 7$ 场比赛。而 ETS 在答案中也设计了这个“迎合部分粗心考生心里”的选项, 让许多人都上了当。

例 4: In a soccer league, if there were 10 teams, and each team played each of the other teams 16 times, how many games did each team play?

- (A) 144
- (B) 45
- (C) 90
- (D) 150
- (E) 160

翻译: 在一个足球联盟中, 假如有 10 支球队, 每支球队要与其他各队踢 16 场球, 那么每支球队踢多少场比赛?

解答: 注意本题问的是 “each team”, 每支球队与其他队比赛 1 次共 9 场 $9 \times 16 = 144$ 所以(A) is correct.

若本题问该联盟共踢多少场比赛, 则应为 $C_{10}^2 \times 16 = 720$

错误分析: 很多同学已经形成了这样的固定思维, 一看到有 10 支球队, 则比赛的场数一定是 $C_{10}^2 = 45$

5.6 计算失误

常见的问题是有些考生把简单的问题复杂化, 结果既做错了题又浪费了时间。也有一些考生没有计算或者粗糙地计算了一下, 便草率做出了决定。

一般情况下, 计算失误出现在 Problem Solving 题中, 但有时也出现在 Data Sufficiency 题中。

许多考生有着错误的认识, 认为所有的 Data Sufficiency 题都不需解出精确的答案。针对这种情况, 本节例举的两个例子都是 Data Sufficiency 题, 并且只有求解后才有可能得出答案。

例 1: If S is the sum of the first n positive integers, what is the value of n ?

- (1) $S < 20$
- (2) $S^2 > 220$

翻译: 假如 S 是前 n 个正整数的和, n 的值是多少?

解答: 由(1) $S < 20$ 无法判断 n 的值, 因为 n 的取值可能有好几个; 由(2) 同样无法得到 n 的值; (1)-(2) 得 $\sqrt{220} < S < 20$, 则 $14 < S < 20$, 只有 $n = 5$ 时才满足上面的不等式。

所以(C) is correct.

错误分析: 不同于很多的 Data Sufficiency 题, 本题只有计算出了 $14 < S < 20$ 才有可能做出正确选择。

有的考生没有计算或者粗糙地计算了一下, 便轻易做出了决定。

提示: 许多 Data Sufficiency 题勿需解出精确答案, 考生不要浪费时间去求解方程。但是也有部分 Data Sufficiency 题, 只有精确求解才有可能得出答案。

例 2: If n is a positive integer and $k = 5.1 \times 10^n$, what is the value of k ?

- (1) $6,000 < k < 500,000$
- (2) $k^2 = 2.601 \times 10^9$

翻译: 假如 n 是一个正整数, 并且 $k = 5.1 \times 10^n$, k 的值是多少?

- (1) $6000 < k < 500000$
- (2) $k^2 = 2.601 \times 10^9$

解答: 由(1)得到 $4 \leq n \leq 4$, 则 $n = 4$, 所以(1)单独回答了问题; 由(2) $k^2 = (5.1)^2 \cdot 10^8$, $n = 4$, 所以(2)也单独回答了问题。

所以(D) is correct.

错误分析: 对于这道 Data Sufficiency 题, 只有精确求解后才有可能得出答案。对于条件 1, 许多考生认为是一个不等式, 即认为不可能求解, 而忽略了对未知数有一些特殊的约束条件: n 是一个正整数。

5.7 其他马虎问题

本节试图概括前几节没有包括的其他一些马虎问题。

(1) 由一些旧有的错误观点派生的错误, 例如一个方程组包括两个方程和两个未知数就一定就能求解。

(2) 不顾公式的使用范围, 简单套用公式。

(3) 主观臆断, 做一些额外的假设, 或者没有将题中给的条件包括进去。

例 1: If $xy = -6$, what is the value of $xy(x + y)$?

$$(1) x - y = 5$$

$$(2) xy^2 = 18$$

翻译: 假如 $xy = -6$, $xy(x+y)$ 的值是多少?

解答: 由(1)中 $x - y = 5$ 与 $xy = -6$ 可以解出 x 、 y 的值, 许多同学认为(1)单独可以回答问题, 其实不然, 因为 $xy = -6$ 是一个二次方程, 解得的 x 和 y 的值各有两个, 所以无法得到 $xy(x + y)$ 的值; (2)中 $xy^2 = 18$ 与 $xy = -6$ 是联立方程, 可得到 y 只有一个值为 -3 , 同理 x 也只有一个值为 2 , 可以求得 $xy(x + y)$ 的值, 所以(B) is correct.

错误分析: 一个方程组包括两个方程和两个

未知数不一定就能准确求解, 还要考虑方程的次数。另外, 还有方程的独立性, 即两个方程之间的关系。例如:

$$3x + 5y = 6$$

$$6x + 10y = 12$$

这个二元一次方程组中两个方程并不是独立的, 而是成比例的, 或者说是等效的, 因此无法求解。

提示: 解方程组时既要考虑方程组的总次数, 还要考虑每个方程之间的关系以及其他限制条件。

例 2: If the sum of the first n positive integers is S , what is the sum of the first n positive even integers, in terms of S ?

$$(A) S/2$$

$$(B) S$$

$$(C) 2S$$

$$(D) 2S - n$$

$$(E) 4S$$

翻译: 假如前 n 个正整数的和为 S , 那么前 n 个正偶数的和是多少, 以 S 表达?

解答: 前 n 个正整数的和为:

$$1 + 2 + 3 + \cdots + n = S$$

前 n 个正偶数的和为:

$$\begin{aligned} 2 + 4 + 6 + \cdots + 2n \\ = 2(1 + 2 + 3 + \cdots + n) \\ = 2S \end{aligned}$$

所以(C) is correct.

错误分析: 解答本题的技巧是: 首先列出两个数列的前几项, 通过比较这两个数列, 我们不难发现第二个数列的每一项都是第一个数列的对应项的 2 倍, 因此它们的和也是第一个数列和的 2 倍。

例 3: If the sum of the first n positive integers is S , what is the sum of the first n positive odd integers, in terms of S ?

$$(A) S/2$$

- (B) S
 (C) $2S$
 (D) $2S-n$
 (E) $4S$

翻译: 假如前 n 个正整数的和为 S , 那么前 n 个的正的奇数的和是多少, 以 S 表达?

解答: 前 n 个正整数的和为:

$$1 + 2 + 3 + \cdots + n = S$$

前 n 个正奇数的和为:

$$1 + 3 + 5 + \cdots + 2n-1 = 2S-n$$

所以(D) is correct.

错误分析: 做完例 2 后有些考生便错误地认为前 n 个的正的奇数的和等于前 n 个正偶数的和, 等于 $2S$ 。

提示: 不要把过多的时间浪费在简单无聊的数学题上面, 最重要的是在做题中学会自己总结。

例 4: When an object is dropped, the number of feet N that it falls is given by the formula $N=1/2gt^2$, where t is the time in seconds since it was dropped and g is 32.2, if it takes 5 seconds for the object to reach the ground, how many feet does it fall during the last 2 seconds?

- (A) 64.4
 (B) 96.6
 (C) 161.0
 (D) 257.6
 (E) 402.5

翻译: 当一个物体被扔下时, 下落的英尺数 N 由公式 $N=1/2gt^2$ 得到, t 是该物体扔下的时间, 以秒为单位, g 为 32.2, 假如花 5 秒钟的时间该物体落下, 在最后两秒它落下了多少英尺?

解答: 5 秒钟该物体共落下的距离为:

$$N=1/2gt^2=1/2 \times 32.2 \times 25=402.5 \text{ 英尺}$$

本题求最后 2 秒下落的距离, 该距离可以通过 5 秒钟下落的距离减去前 3 秒下落的距离:

$$402.5-(1/2)gt^2=402.5-(1/2) \times 32.2 \times 9 \\ =257.6 \text{ 英尺}$$

所以(D) is correct.

错误分析: 本题中给出一个公式, 有的考生简单地将 2 秒代入这个公式:

$$N=1/2gt^2=1/2 \times 32.2 \times 4=64.4 \text{ 英尺}$$

提示: 使用公式一定要注意公式的使用范围。

例 5: Can the relationship of $2n + r$ and $2s + t$ be determined?

$$(1) n/4 + r/8 = s/3 + t/6$$

(2) n, r, s, t are positive integers

翻译: $2n + r$ 和 $2s + t$ 的大小关系能决定吗?

$$(1) n/4 + r/8 = s/3 + t/6$$

(2) n, r, s, t 是正整数

解答: 单独的条件 1 和条件 2 都无法回答本题, 如果把条件 1 和条件 2 结合起来:

$$\text{设 } n/4 + r/8 = x = s/3 + t/6$$

$$\text{那么 } 2n - r = 8x \text{ 而 } 2s + t = 6x$$

上面都是正整数, $8x > 6x$

所以(C) is correct.

错误分析: 这是一道易粗心的题, 考生忽略了条件 2, 从而错误地选择了选项 E。

5.8 本章习题精选

- If a motorist had driven 1 hour longer on a certain day and at an average rate of 5 miles per hour faster, he would have covered 70 more miles than he actually did. How many more miles would he have covered than he actually did if he had driven 2 hours longer and at an average rate of 10 miles per hour faster on that day?
 - 100
 - 120
 - 140
 - 150
 - can not be determined
- If today the price of an item is \$3,600, what was the price of the item exactly 2 years ago?
 - The price of the item increased by 10 percent per year during this 2-year period.
 - Today the price of the item is 1.21 times its price exactly 2 years ago.
- By what percent has the price of an overcoat been reduced?
 - The original price was \$380.
 - The original price was \$50 more than the reduced price.
- Was 70 the average (arithmetic mean) grade on a class test?
 - On the test, half of the class had grades below 70 and half of the class had grades above 70.
 - The lowest grade on the test was 45 and the highest grade on the test was 95.
- A jewelry dealer initially offered a bracelet for sale at an asking price that would give a profit to the dealer of 40 percent of the original cost. What was the original cost of the bracelet?
 - After reducing this asking price by 10 percent, the jewelry dealer sold the bracelet at a profit of \$403.
 - The jewelry dealer sold the bracelet for \$1,953.
- Who types at a faster rate, John or Bob?
 - The difference between their typing rates is 10 words per minute.
 - Bob types at a constant rate of 80 words per minute.
- If a rope is cut into three pieces of unequal length, what is the length of the shortest of these pieces of rope?
 - The combined length of the longer two pieces of rope is 12 meters.
 - The combined length of the shorter two pieces of rope is 11 meters.
- If the price of a magazine is to be doubled, by what percent will the number of magazines sold decrease?
 - The current price of the magazine is \$1.00.
 - For every \$0.25 of increase in price, the number of magazines sold will decrease by 10 percent of the number sold at the current
- A certain type of concrete mixture is to be made of cement, sand, and gravel in a ratio 1:3:5 by weight. What is the

- greatest number of kilograms of this mixture that can be made with 5 kilograms of cement?
- (A) $13\frac{1}{2}$
 (B) 15
 (C) 25
 (D) 40
 (E) 45
10. Audrey went shopping with D dollars. She spent 20 percent of her money on a blouse and 25 percent of what was left on a pair of shoes. What percent of the original D dollars did she spend?
- (A) 25%
 (B) 40%
 (C) 45%
 (D) 47%
 (E) 50%
11. A, B, and C each drove 100-mile legs of a 300-mile course at speeds of 40, 50, and 60 miles per hour, respectively. What fraction of the total time did A drive?
- (A) $\frac{15}{74}$
 (B) $\frac{4}{15}$
 (C) $\frac{15}{37}$
 (D) $\frac{3}{5}$
 (E) $\frac{5}{4}$
12. In a student body the ratio of men to women was 1 to 4. After 140 additional men were admitted, the ratio of men to women became 2 to 3. How large was the student body after the additional men were admitted?
- (A) 700
 (B) 560
 (C) 280
 (D) 252
 (E) 224
13. Of the following, which best approximates $\frac{(0.1667)(0.8333)(0.3333)}{(0.2222)(0.6667)(0.1250)}$?
- (A) 2.00
 (B) 2.40
 (C) 2.43
 (D) 2.50
 (E) 3.43
14. How many of the integers between 25 and 45 are even?
- (A) 21
 (B) 20
 (C) 11
 (D) 10
 (E) 9
15. A plane traveled k miles in the first 96 minutes of flight time. If it completed the remaining 300 miles of the trip in t minutes, what was its average speed, in miles per hour, for the entire trip?
- (A) $\frac{60(k+300)}{96+t}$
 (B) $\frac{kt+96(300)}{96t}$
 (C) $\frac{k+300}{60(96+t)}$
 (D) $\frac{5k}{8} \frac{60(300)}{t}$
 (E) $\frac{5k}{8} + 5t$

16. A milliner bought a job lot of hats, $\frac{1}{4}$ of which were brown. The milliner sold $\frac{2}{3}$ of the hats including $\frac{4}{5}$ of the brown hats. What fraction of the unsold hats were brown?
- (A) $\frac{1}{60}$
(B) $\frac{2}{15}$
(C) $\frac{3}{20}$
(D) $\frac{3}{5}$
(E) $\frac{3}{4}$
17. At a special sale, 5 tickets can be purchased for the price of 3 tickets. If 5 tickets are purchased at this sale, the amount saved will be what percent of the original price of the 5 tickets?
- (A) 22%
(B) $33\frac{1}{3}\%$
(C) 40%
(D) 60%
(E) $66\frac{2}{3}\%$
18. Working independently, Tina can do a certain job in 12 hours. Working independently, Ann can do the same job in 9 hours. If Tina works independently at the job for 8 hours and then Ann works independently, how many hours will it take Ann to complete the remainder of the job?
- (A) $\frac{2}{3}$
(B) $\frac{3}{4}$
(C) 1
(D) 2
(E) 3
19. Three automobiles travel distances that are in the ratios of 1:2:3. If the ratios of the traveling times over these distances for these automobiles are 3:2:1 in the same respective order, what are the ratios of their respective average speeds?
- (A) 1:1:1
(B) 1:2:3
(C) 1:3:9
(D) 3:2:1
(E) 3:4:3
20. Over the last three years a scientist had an average (arithmetic mean) yearly income of \$45,000. The scientist earned $1\frac{1}{2}$ times as much the second year as the first year and $2\frac{1}{2}$ times as much the third year as the first year. What was the scientist's income the second year?
- (A) \$9,000
(B) \$13,500
(C) \$27,000
(D) \$40,500
(E) \$45,000

21. At a Wall Street company, 70 percent of this year's new employees are graduates of business schools and the remainders are graduates of liberal arts colleges. If 550 new employees were hired this year, what is the difference between the number of new business school employees and the number of new liberal arts employees?
- (A) 55
(B) 220
(C) 240
(D) 385
(E) 440

参考答案

1~5	DDCED
6~10	EECEB
11~15	CBDDA
16~20	CCECD
21~22	BB

22. In performing a sequence of experiments, a scientist made 20 measurements. The average (arithmetic mean) of these measurements was 34. For security reasons the scientist coded the data by multiplying each of the measurements by 10 and then adding 40 to each product. What is the average of the coded measurements?
- (A) 1,140
(B) 380
(C) 342
(D) 57
(E) 19

第6章 思维陷阱

我国考生的一个共同特点是计算能力相对较强，而逻辑思维能力较弱。这一共性使得我国考生往往在 Data Sufficiency 这一部分的得分比较低。

本章总结的是由于思维陷阱对考生造成的失分，主要针对的题型是 Data Sufficiency 题。通过本章的学习，让读者能够尽量克服自己固有的一些逻辑思维的错误，在 GMAT 数学和 Data sufficiency 部分获得高分。

6.1 回答是No

Data Sufficiency 题目主要考查是否能够回答问题。在“answer the question”时，不管是“Yes”还是“No”都是一种回答。只要能够回答就是答案。许多考生潜意识里只将“Yes”的回答当作回答了问题。

例 1: Is x equal to 2?

(1) $x^2 = 2$

(2) $x^2 - x - 1 = 0$

翻译: 题目中 $x = 2$ 吗?

解答: 从(1)得 $x = \sqrt{2}$ or $x = -\sqrt{2}$ 。即单独(1)能够回答问题，回答是“No”。同理，单独(2)也能够回答问题，回答是“No”。所以(D) is correct。

提示: 只将“Yes”的回答当作回答了问题，是造成众多考生失分的主要原因之一。

例 2: In $\triangle PQR$ above, is $PQ > PR$?

(1) $x = y$

(2) $y = z$

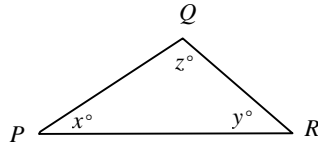


图 6.1

翻译: 如图 6.1 的 $\triangle PQR$ 中, $PQ > PR$?

解答: (1)中 $x = y$ 无法做出判断, 而(2)中 $y = z$, 则得到 $PQ = PR$, 所以本题(B) is correct, 因为对上面的问题回答是 “No”。注意: 许多考生习惯于认为只有 “Yes” 才算回答了问题, “No” 同样也是回答。

例 3: Each person on a committee with 40 members voted for exactly one of 3 candidates, F, G, or H. Did Candidate F receive the most votes from the 40 votes cast?

(1) Candidate F received 11 of the votes.

(2) Candidate H received 14 of the votes.

翻译: 在有 40 名成员的委员会中, 每个人都对 3 名候选人 F、G 或 H 中的一个投票支持。候选人 F 接受了 40 张选票中的大多数吗?

(1) 候选人 F 接受了 11 张选票。

(2) 候选人 H 接受了 14 张选票。

解答: 由(1)F 只接受了 11 张选票, 则对上面的问题的回答为 “No”, 所以(1)单独回答了上面问题(注意: No 也是一种回答); (2)中无法看出 F 怎么样, 所以(A) is correct。

提示: 相对于例 1 和例 2, 本题的隐蔽性更强。很多考生无法从条件(1)中推断出

候选人 F 接受的选票肯定不是最多。也有的考生虽然得出了候选人 F 接受的选票肯定不是最多,由于固有的错误思维,仍选择了错误的选项 E。

6.2 答案唯一

GMAT 官方的 Directions 关于值的唯一性是这么描述的: In data sufficiency problems that ask for the value of a quantity, the data given in the statements are sufficient only when it is possible to determine exactly one numerical value for the quantity. 也就是说这个值必须惟一,如果从条件确定出来有两个值或者两个以上值,回答都是不充分的。

例 1: Is x equal to 2?

(1) $x^2 = 4$

(2) $x^2 - x - 2 = 0$

翻译: 题目中 $x = 2$ 吗?

解答: 从(1)得 $x = 2$ or $x = -2$ 。单独(1)能够回答问题吗?不能!问一个量的值是多少,这个值必须是惟一的。现在 X 有两个值,当然不能够回答问题。同理,单独(2)得到 $x = -1$ or $x = 2$,单独(2)也不能回答问题。把(1)和(2)结合起来得到 $x = 2$,那么就能够回答问题了。

所以(C)is correct。

提示: 像“题目中 $x = 2$ 吗?”,实际上问的是“ x 一定等于 2 吗?”,而不是像某些人理解的“ x 可能等于 2 吗?”。

例 2: Pam and Ed are in a line to purchase tickets. How many people are in the line?

(1) There are 20 people behind Pam and

20 people in front of Ed.

(2) There are 5 people between Pam and Ed.

翻译: P 和 E 排队买票,该队列中有多少人?

(1) 有 20 个人在 P 的后面且有 20 个人在 E 的前面。

(2) P 和 E 之间有 5 个人。

解答: 由(1)无法得到该队列的人数,因为不知道 P 和 E 之间有多少人;由(2)也无法回答上面的问题;(1)+(2)同样无法回答上面的问题,因为不知道 P 和 E 谁在前面,若 P 在前面,则该队列有 $20+20-5=35$ 人,若 E 在前面,则该队列有 $20+20+2-5=47$ 人。

所以(E)is correct。

错误分析: 由于理解或者思维的原因,有的考生认为 E 站在 P 的前面,因此该队列的人数可以确定。

例 3: A, B, and C are points on a line. Can the relationship of the distance between A and B and 30 be determined?

(1) The distance between A and B is twice the distance between A and C.

(2) The distance between C and B is 10.

翻译: A、B、C 是一条直线上的点。A 和 B 之间的距离与 30 的大小关系能够确定吗?

(1) A 和 B 之间的距离是 A 和 C 之间距离的 2 倍。

(2) C 与 B 的距离是 10。

解答: 本题存在两种情况,如图 6.2 所示。

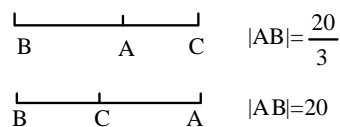


图 6.2

A 与 B 之间的距离是 20 或者 $20/3$, 无

论哪种情况都比 30 小。

所以(C)is correct。

错误分析: 很多考生都考虑到两种情况, 根据“答案惟一”的原则, 他们便认定大小关系无法确定。这实际上是对“答案惟一”原则的曲解。因为本题问的是大小关系, 而不是确切的值。

提示: 本书的目的是提供 GMAT 数学中一些常用的原则、方法和技巧, 读者可在理解的基础上灵活应用, 切勿生搬硬套。

6.3 两个条件的独立性

解 Date Sufficiency 题目的步骤是首先独立考虑两个条件, 在考虑一个条件时绝对不能受另一个条件的影响。许多考生在考虑条件(2)时候, 潜意识中已经将条件(1)当成了题中所给的一个条件, 从而错误地选择答案。

如果一个条件已经能够独立回答问题, 而另外一个条件不能, 就绝对没有必要再把这两个条件联合起来求解。

例 1: If S is an infinite set of real numbers, is there a number in S that is less than every other number in S ?

(1) Every number in S is an integer.

(2) Every number in S is positive.

翻译: 假如 S 是一个实数的无限集合, S 中有数字小于 S 中的其他每个数吗?

(1) S 中的每个数字都是整数。

(2) S 中的每个数字都是正数。

解答: 由(1)无法得到任何结果, 因为 S 中的数可以为负数; 由(2)也无法回答上面的问题, 注意 0 既不是正数也不是负数; (1)+(2)可以回答上面的问题, 因为 S 中可能有的最小的一个数是 1。

所以(C)is correct。

错误分析: 有人在考虑条件(2)时, 潜意识中已经认为“ S 中的每个数字都是整数”, 因此错误地选择了选项(B)。

例 2: How many integers n are there such that $r < n < s$?

(1) $s - r = 5$

(2) r and s are not integers.

翻译: 有多少正整数 n 满足 $r < n < s$?

(1) $s - r = 5$

(2) r 和 s 不是整数。

解答: (1)中 $s - r = 5$, 若 s, r 为整数, 则有 4 个正整数满足 $r < n < s$, 若 s, r 为小数, 则有 5 个正整数满足 $r < n < s$, 所以(1)无法回答上述问题; (2) r 和 s 不是整数也不能回答上述问题; (1)+(2)可以得到有 5 个正整数满足: $r < n < s$

所以(C)is correct。

错误分析: 有人在考虑条件(2)时, 潜意识中已经认为“ $s - r = 5$ ”, 因此错误地选择了选项(B)。

提示: 考生应该牢记两个条件是独立的, 在考虑一个条件时绝对不能受另一个条件的影响, 尤其是考虑条件(2)的时候不能受条件(1)的影响。

例 3: Is x equal to 2?

(1) $x^2 = 4$

(2) $(x - 2)^2 = 0$

翻译: 题目中 $x = 2$ 吗?

解答: 从(1)得 $x = 2$ or $x = -2$ 。单独(1)能够回答问题吗? 不能! 问一个量的值是多少, 这个值必须是惟一的。现在 x 有两个值, 当然不能够回答问题。单独(2)就能得到 $x = 2$, 所以仅(2)就能够回答问题。就没有必要把(1)和(2)结合起来

考虑了。

所以(B) is correct.

提示: 如果一个条件已经能够独立回答问题, 而另外一个条件不能, 就绝对没有必要再把这两个条件联合起来求解。

例 4: Are all of the numbers in a certain list of 15 numbers equal?

- (1) The sum of all of the numbers in the list is 60.
- (2) The sum of any 3 numbers in the list is 12.

翻译: 某一数列中的 15 个数字都相等吗?

- (1) 该数列中所有数字的和是 60。
- (2) 该数列中任 3 个数字的和是 12。

解答: 由(1)中所有数字的和为 60 无法得到其中 15 个数字是否都相等。(2)由任 3 个数的和都为 12, 则要满足这一条件, 要求这 15 个数都是 4, 所以由(2)可以回答这个问题。

所以(B) is correct.

错误分析: 条件(2)单独可以回答这个问题, 有人认为条件(1)和条件(2)结合起来也可以回答问题, 选择了答案 C, 忽略了两个条件是相互独立的。

例 5: What is the value of x ?

- (1) $x > 45$
- (2) $x < 45$

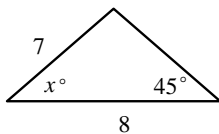


图 6.3

翻译: 如图 6.3 所示, x 的值是多少?

解答: 由 x 角的顶点与对边引一条垂线得到一个直角, 则由一个角为 45° 可得这个直角边长度为 $4\sqrt{2} < 7$, 由此可以取

出两个长度为 7 的边, 一个与长度为 8 的底边所成角度大于 45° , 另一个小于 45° , 如图 6.4 所示。

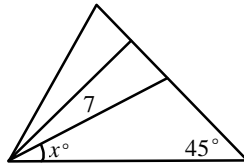


图 6.4

无论是 $x > 45$ 还是 $x < 45$, x 的值都是确定的。

所以(D) is correct.

6.4 二元一次方程组

如果一个一次方程组有两个未知数和两个方程, 那么这个方程组就一定能够求解吗? 很多考生毫不犹豫地回答“是”。

正是这种错误的逻辑导致了我国考生在 GMAT 数学中不能获得高分。二元一次方程组能否求解, 取决于两个方程是否独立。如果两个方程不独立(成比例或者说是等效), 这个方程组就无法求解。

一个方程有两个未知数, 如果加上其他限制条件(如未知数必须是整数等), 方程也可能求解。

例 1: A haberdasher(男子服饰经销商)sells neckties for \$ 12 each and shirts for \$ 12 each. If he sells \$ 96 worth of ties and shirts and the total number of ties and shirts is 8, what is the amount of ties he sells?

- (A) 3
- (B) 4
- (C) 5
- (D) 6
- (E) Can not be determined

翻译：一家男子服饰经销商出售领带和衬衫，价格均为每件 12 美元。如果他卖了 8 件东西，总共卖了 96 美元，问他卖了多少条领带？

解答：设卖了 x 条领带， y 件衬衫。有人认为即可得一个二元一次方程组，因此可以求解。实际上两个方程分别为：

$$12x + 12y = 96$$

$$x + y = 8$$

这个二元一次方程组中两个方程并不是独立的，而是等效的，因此无法求解。所以(E)is correct。

提示：如果二元一次方程组有两个方程，只有这两个方程是独立的才能求解。

例 2：What is Steve's annual salary and Maria's annual salary?

(1) The combined total of the annual salaries of Steve and Maria is \$ 80,000.

(2) If both Steve and Maria were to receive a 10 percent increase in annual salary, their combined annual salaries would be \$ 88,000.

翻译：斯蒂芬和玛丽娅的年薪各是多少美元？

(1) 斯蒂芬和玛丽娅的年薪总共为 80,000 美元。

(2) 如果斯蒂芬和玛丽娅的年薪都增加 10%，他们的年薪总共为 88,000 美元。

解答：设斯蒂芬和玛丽娅的年薪分别为 X 和 Y 。根据(1)只能列出一个方程： $X+Y=80,000$ (不必写出)，一个方程无法解出两个未知数，所以(1)不充分。根据(2)也只能列出一个方程： $1.1X+1.1Y+88,000$ ，所以(2)也不充分。(1)和(2)相结合形成二元一次方程组，

由于这两个方程并不是独立的，也无法求出 X 和 Y 的值。

所以(E)is correct。

提示：在解 Data Sufficiency 题时，尽管考生不要浪费时间去寻找问题的答案，但是应该注意数字之间的关系。

例 3：If x and y are positive integers, what is the value of y ?

(1) $21x + 20y = 480$

(2) $x + y = 24$

翻译：如果 x 和 y 都是正整数，那么 y 的值是多少？

解答：这道题有很大的迷惑性，有些考生不加思考就选择(C)，但是这样做就忘了 x, y 都是正整数的限制。单独(1)得到 $y=24-\frac{21}{20}x$ ，由于 20 和 21 互质，那么 x 必然是 20 的倍数，但是由于 y 也是正整数，所以必然取 20，这样 y 的值也就确定。

所以(A)is correct。

提示：在某些人看来，两个未知数通过一个方程都获得了解似乎是不可思议的。如果加上其他限制条件(如未知数必须是整数等)，方程也可能求解。本题即是一例。

例 4： $14r+s=86$, s, r 均为自然数，求 $r+s=?$

(1) $s < 16$

(2) $r > 5$

解答：将原式变形为： $s = 14(6-r) + 2$

即 s 除以 14 余 2，根据条件(1)得：

$s=2, r=6$ 。同理由条件(2)可得： $s=2, r=6$ 。

所以(D)is correct。

提示：本题通过不等式的限制条件去求解二元一次方程。

6.5 排列组合问题

本节总结的是考生在排列组合问题中常犯的逻辑错误。许多考生忽略了题目要求，没有将不同情况分类，只考虑了其中一种或几种，将题目简单化。

例 1: A company that ships boxes to a total of 15 distribution centers uses color coding to identify each center. If either a single color or a pair of two different colors is chosen to represent each center and if each center is uniquely represented by that choice of one or two colors, what is the minimum number of colors needed for the coding? (Assume that the order of the colors in a pair does not matter.)

- (A) 4
- (B) 5
- (C) 6
- (D) 7
- (E) 15

翻译: 运送盒子到 15 个分配中心的公司使用颜色码来确定每个中心。假如一个颜色或者一对不同的颜色来代表一个中心，并且假如每个公司所用的一个或一对颜色是独特的，最少需要多少种颜色来标记这些分配中心？(假如每一对颜色中颜色的顺序不会造成区别)

解答: 每个分配中心可用一种或两种颜色且与其他的分配中心进行区分，显然若每个公司只用一种颜色则需要 15 种不同的颜色，但如果有的公司用两种颜色，可以通过两种颜色的不同组合来区别每个分配中心。采用排除法，(A)4 种颜色无论如何不足以区别 15 个分配中心；(B)5 种颜色可以单独表示 5 个分配中心，而从中任取两种颜色 $C_5^2=10$ ，

两者相加足以表示 15 个分配中心。

所以(B)is correct。

错误分析: 有人把本题理解为一个简单的排列组合问题：用两种颜色的不同组合来区别 15 个分配中心，至少需要多少种颜色。由于 $C_6^2=15$ ，他们选择了选项 (C)。

例 2: Are there exactly 3 distinct symbols used to create the code words in language Q?

- (1) The set of all code words in language Q is the set of all possible distinct horizontal arrangements of one of more symbols, with no repetition.
- (2) There are exactly 15 code words in language Q.

翻译: 有 3 个不同的符号用于创造出语言 Q 中的密码吗？

- (1) 语言 Q 中的所有密码单词的集合是一个或多个符号的所有可能的水平排列，且没有重复。
- (2) 语言 Q 中共有 15 个密码单词。

解答: 由(1)无法判断是否有 3 个不同的符号，(2)语言 Q 中有 15 个密码单词，但不知道这些密码单词是否可以纵向排列或斜向排列，以构成不同的密码单词，所以(2)单独无法回答问题；(1)+(2)可以回答问题，因为 3 个字母正好可以构成 15 个水平排列的密码单词，1 个字母的有 3 个，2 个字母的有 P_3^2 个，3 个字母的有 P_3^3 个，所以共 15 个。

所以(C)is correct。

错误分析: 与例 1 很相似，密码并不局限于由 3 个字母组成，而是可以由 1、2 或 3 个字母的排列组成。

例 3: 6 个人分成 3 组，每组 2 人，有多少

种方法?

解答: 注意本题和下面这个问法的区别:

6个人分成A, B, C 3组, 每组2人, 有多少种方法?

由于3个组是有区别的, 因此分法数:
 $C_6^2 C_4^2 = 90$ 。

而在本题中3个小组是没有区别的, 因此分法数:

$$\frac{C_6^2 C_4^2 C_2^2}{P_3^3} = 15$$

提示: 分组问题要注意组是有差别的还是无差别的。

6.6 数论

数论部分的思维陷阱可以概括为“等不等”、“正不正”、“整不整”等。下面举例详细说明。

例 1: 若 a, b 均为大于 250 的自然数, 求 a, b 最小公倍数的最小可能值。

解答: a, b 均为大于 250 的自然数, a, b 可以相等也可以不相等, 若 $a=b=251$, 则 a, b 的最小公倍数为 251。若 a, b 为其他值时, a, b 的最小公倍数一定会大于 251。

错误分析: 很多考生潜意识中认为 a, b 不相等, 不少人都没有解题思路, 也有一些考生认为 a (或 b)=251, b (或 a)=251×2=502, a, b 的最小公倍数为 502。

提示: 不要认为两个不同字母所代表的两个数字就不能相等, 这就是“等不等”的陷阱。

例 2: A computer is programmed to generate a list of multiples of prime numbers 2, 3 and 5 as shown below:

Program 1 – List multiples of 2

Program 2 – List multiples of 3

Program 3 – List multiples of 5

图 6.5

How many integers less than 100 will appear on all of the lists of programs produced above?

(A) None

(B) 1

(C) 3

(D) 5

(E) An infinite number of integers

翻译: 一个计算机被编程为产生一组质数 2, 3, 5 的倍数。如图 6.5 所示: 程序 1 列出 2 的倍数, 程序 2 列出 3 的倍数, 程序 3 列出 5 的倍数。

有多少小于 100 的整数将出现在上面程序所产生的所有数列中?

解答: 本题问的是在小于 100 的整数中有多少数同时是 2, 3, 5 的倍数。小于 100 的整数包括整数数和负整数, 2、3、5 的公倍数(即 30 的倍数)有无穷多个。所以(E)is correct。

错误分析: 受习惯思维的影响, 很多人认为倍数, 公倍数等概念只局限于正整数, 错误地把本题等同于“小于 100 的正整数中有多少数同时是 2,3,5 的倍数。”

提示: 本题是“正不正”陷阱问题中的一个很好的例子。

例 3: $a-b=5$, 求 a, b 之间的整数的个数。

(1) a, b 都是整数

(2) a, b 都不是整数

解答: 条件(1), 若 a, b 都是整数, 则 a, b 之间的整数的个数为 4。如 $a=6, b=1$, a, b 之间有 2、3、4、5 这四个整数。条件(2), 若 a, b 都不是整数, 则 a, b 之间的整数的个数为 5。如 $a=6.1, b=1.1$, a, b 之间有 2、3、4、5、6 这

五个整数。

所以(D) is correct.

提示: 只有分别讨论“整不整”，本题才有可能求解。

例 4: 已知 $-1 < x < 1$ 且 $x \neq 0$, m, n 均为正整数, 且 $m > n$, 比较 x^m 与 x^n 的大小。

(1) m, n 均为奇数

(2) m, n 均为偶数

解答: 条件(1), 若 $-1 < x < 0$, 则 $x^m > x^n$;
若 $0 < x < 1$, 则 $x^m < x^n$ 。条件(1)单独无法判断。

条件(2), 无论 $-1 < x < 0$ 还是 $0 < x < 1$, 都有 $x^m < x^n$ 。

所以(B) is correct.

错误分析: 当 $0 < x < 1$ 时, 幂随指数的增大而减小, 即 $m > n > 0$, $x^m < x^n$ 。

当 $-1 < x < 1$ 时, 幂的绝对值随指数的增大而减小, 即 $m > n > 0$, $|x^m| < |x^n|$ 。

很多人混淆了这两条性质, 简单地认为: 当 $-1 < x < 1$ 时, 幂随指数的增大而减小。他们认为无论奇偶都没有区别, 所以选择了选项(D)。

提示: 负数的幂的运算, 首先应该考虑指数的奇偶问题, 即“偶不偶”。

6.7 概率

概率问题中的思维陷阱常常表现在两个方面: 一是相关性, 即考生需要明确所求的概率是一个新的事件, 还是原有事件的组合, 新的事件不可能求解; 二是独立性, 只有事件 A 和 B 是相互独立的, 才能用概率公式 $P(AB) = P(A)P(B)$ 计算。

例 1: 求 A、B 都发生的概率。

(1) A 发生的概率是 0.6

(2) B 发生的概率是 0.7

解答: 很多人都选择了答案 C, 并且还坚持自己用的是概率公式:

$$P(AB) = P(A)P(B)$$

其实这个公式的前提条件是: 事件 A 和 B 是相互独立的。本题没有告诉 A 和 B 是否相互独立, 所以(E) is correct.

提示: 对于多个事件的概率问题, 首先应该清楚各个事件之间是否相互独立。

例 2: 事件 A 发生的概率是 0.6, 事件 B 发生的概率是 0.7。求 A、B 都不发生的最大概率。

(A) 0.12

(B) 0.3

(C) 0.4

(D) 0.88

(E) 无法确定

解答: 由于题目中没有说事件 A 和 B 是相互独立的, 因此不能用概率公式 $P(AB) = P(A)P(B)$ 计算。

A 不发生的概率是 0.4, B 不发生的概率是 0.3, 它们的最大交集是 0.3。

所以(B) is correct.

例 3: A 独立完成某项任务成功的概率是 0.6, B 独立完成某项任务成功的概率是 0.7, 求 A、B 一起去完成, 则不成功的概率是多少?

(A) 0.12

(B) 0.3

(C) 0.4

(D) 0.88

(E) 无法确定

解答: “A、B 一起去完成”是一个新事件, 与 A 和 B 独立完成某项任务成功的概率没有关系, A 和 B 可能相互促进, 也可能相互排斥。

所以(E) is correct.

错误分析：很多人认为本题与例 2 没有区别，实际上是对事件的独立性理解还不够深刻。

6.8 主观假设

除了前几节谈到的“回答是 No”、“答案惟一”、“两个条件的独立性”等，我国考生经常犯的一个错误是主观臆断，除了题目所提供的信息和一些永恒不变的事实，额外做一些的假设。下面就是几个很好的例子。

例 1: How many minutes long is time period X?

- (1) Time period X is 3 hours-long.
- (2) Time period X starts at 11 p.m and ends at 2 a.m.

翻译：时间区 X 一定有多少分钟？

解答：(1)显然可以回答问题，但问题在于(2)。(2)是否可以回答问题呢？不一定，因为不知道开始和结束是否在同一天。所以(A)is correct。

错误分析：很多考生主观上已经假设“开始和结束是在同一天”，因此选择了选项(D)。

提示：考生只应该根据题目和所提供的信息、一些永恒不变的事实(如每天 24 小时)进行答题。切勿主观臆断，做一些额外的假设。

例 2: If a certain grove consists of 36 pecan trees, what was the yield per tree last year?

- (1) The yield per tree for the 18 trees in the northern half of the grove was 60 kilograms last year.
- (2) The yield per tree for the 18 trees in the eastern half of the grove was 55

kilograms last year.

翻译：假如某一小树林中有 36 棵山核桃树，去年每棵树的平均产量是多少？

- (1) 去年小树林北半部的 18 棵树每棵树的平均产量是 60 千克。
- (2) 去年东半部的 18 棵树每棵树的平均产量是 55 千克。

解答：由(1)无法得到整个小树林的平均产量；同理(2)也无法得到小树林的平均产量，(1)+(2)也同样无法得到结果。所以(E)is correct。

错误分析：很多“聪明”的考生都选择了答案 C。他们犯了两个错误：一是没有认真读题，认为一个条件描述的是“小树林的北(西)半部”，另外一个条件描述的是“小树林的南(东)半部”；二是主观上认为小树林由南北(或者东西)两部分组成。

例 3: At 9 a.m, a hiker was due south of point P. What direction was point P from her position at noon?

- (1) From 9 a.m until 11 a.m she walked due east at 2 miles per hour, and from 11 a.m until noon, she walked due north at 3 miles per hour.
- (2) At noon, she is exactly 4.5miles from point P.

翻译：上午 9 点，一个徒步旅行者在点 P 的正南方，正午时，点 P 在她的哪个方向？

- (1) 从上午 9 点到上午 11 点，她向正东以每小时两英里的速度行走，并且从上午 11 点后直到中午，她以每小时 3 英里的速度向北行走。
- (2) 在中午，她距离 P 点 4.5 英里。

解答: at noon 指正午 12 点, 由(1)她向东走了 4 英里后又向北走了 3 英里, 但由于未知上午 9 点时其距 P 点的距离所以无法得到此时她在 P 点的哪个方向; (2) 正午时距 P 点 4.5 英里无法得到其距 P 点的方向; (1)+(2)同样无法确定其距 P 点的方向。

所以(E)is correct。

错误分析: 许多考生都被这道题所迷惑, 选择了答案 A, 因为他们潜意识中假设了“开始从 P 点出发”。

例 4: What fractional part of the total surface area of cube C is red?

(1) Each of 3 faces of C is exactly 1/2 red.

(2) Each of 3 faces of C is entirely white.

翻译: 立方体 C 的红色部分占了该立方体表面积的比例是多少?

(1) C 的 3 个面的每个面有一半是红色。

(2) C 的 3 个面的每个面都是完全的白色。

解答: 由(1)无法回答上面的问题, 因为不知道另外 3 个面是什么颜色; 由(2)同理不知道另外 3 个面的颜色; (1)+(2)可以求得该比例, 因为 6 个面的情况都已知。

所以(C)is correct。

错误分析: 如果本题换成了“What fractional part of the total surface area of cube C is white?”, 答案就是 E, 因为没有什么地方提示正方体只用红和白两种颜色。

例 5: The table above shows the morning schedule for train X, If Juan took train X on Monday morning, did he arrive at station T on schedule?

(1) Juan arrived at station T on Monday morning 1 hour and 2 minutes after

he left station S.

(2) Juan arrived at his office at 8:30(EST) on Monday morning, which was 20 minutes after he arrived at station T.

翻译: 图 6.6 所示的是火车 X 早晨的时间安排, 假如 J 在星期一早晨乘坐火车 X, 他能准时到达车站 T 吗?

DAILY TRAIN SCHEDULE		
Train	Scheduled	Scheduled
	Departure	Arrival
	Station S	Station T
X	7:08(EST)*	8:10(EST)
* Eastern Standard Time		

图 6.6

(1) J 星期一早晨在离开车站 S 后的 1 小时 2 分钟后到达车站 T。

(2) J 在星期一早晨 8:30(东部标准时间)到达他的办公室, 这个时间是他到达车站 T 后的 20 分钟。

解答: 本题问 J 是否准时到达车站 T, (1)中指出火车行驶了 1 小时 2 分钟, 正是时间安排中的行驶时间, 但是该火车是否准时从 S 出发是未知的, 所以是否准时到达 T 也就未知了, (1)无法单独回答问题; (2)中指出 J 在 8:30 到达办公室, 是其到达车站 T 的 20 分钟后, 则 J 到达 T 的时间为 8:10, 准时到达, 仅(2)单独回答了上面问题。

所以(B)is correct。

提示: 除了题目和所提供的信息, 考生不应该有一些额外的假设。本题中火车的行驶时间等于时间安排中的行驶时间, 考生若认定火车能够准时到达 T, 必须假设该火车准时从 S 出发。而本题中这一信息是未知的。

6.9 其他思维陷阱

例 1: 2000 年初 A、B、C 三人合伙建立一个公司，三人分别投入 \$20,000，\$30,000，\$40,000。2000 年底 A、B、C 根据全年投资总额分配利润。求 A 所得利润。

(1) 总利润为 \$20,000。

(2) A、B、C 每次的投资都和最初投资成比例。

解答: 本题有两个不同的概念，一是年初投入量，二是全年投资总额。条件(2)告诉我们每次的投入量都与最初投资成比例，即等于三人全年投资总额的比例。条件(1)和条件(2)结合起来即可求出 A 所得利润。

所以(C)is correct。

提示: 很多 Data Sufficiency 题中问题和条件都存在差异，能否明确这种差异是解题的关键。

6.10 本章习题精选

1. Town T has 20,000 residents, 60 percent of whom are female. What percent of the residents were born in Town T?

(1) The number of female residents who were born in Town T is twice the number of male residents who were not born in Town T.

(2) The number of female residents who were not born in Town T is twice the number of female residents who were born in Town T.

2. Is William taller than Jane?

(1) William is taller than Anna.

(2) Anna is not as tall as Jane.

3. What was John's average driving speed in miles per hour during a 15-minute interval?

(1) He drove 10 miles during this interval.

(2) His maximum speed was 50 miles per hour and his minimum speed was 35 miles per hour during this interval.

5. At a certain university, if 50 percent of the people who inquire about admission policies actually submit applications for admission, what percent of those who submit applications for admission enroll in classes at the university?

(1) Fifteen percent of those who submit applications for admission are accepted at the university.

(2) Eighty percent of those who are accepted send a deposit to the university.

6. What is the value of x ?
- (1) $x + y = 7$
 - (2) $x - y = 3 - y$
7. If positive integer x is divided by 2, the remainder is 1. What is the remainder when x is divided by 4?
- (1) $31 < x < 35$
 - (2) x is a multiple of 3.
8. What was Bill's average (arithmetic mean) grade for all of his courses?
- (1) His grade in social studies was 75, and his grade in science was 75.
 - (2) His grade in mathematics was 95.
9. If $S = \{2, 3, x, y\}$, what is the value of $x + y$?
- (1) x and y are prime numbers.
 - (2) 3, x , and y are consecutive odd integers in ascending order.
10. Claire paid a total of \$1.60 for stamps, some of which cost \$0.20 each, and the rest of which cost \$0.15 each. How many 20-cent stamps did Claire buy?
- (1) Claire bought exactly 9 stamps.
 - (2) The number of 20-cent stamps Claire bought was 1 more than the number of 15-cent stamps she bought.
11. If John is exactly 4 years older than Bill, how old is John?
- (1) Exactly 9 years ago John was 5 times as old as Bill was then.
 - (2) Bill is more than 9 years old.
12. Before play-offs, a certain team had won 80 percent of its games. After play-offs, what percent of all its games had the team won?
- (1) The team competed in 4 play-off games.
 - (2) The team won all of its play-off games.
13. Last Tuesday a trucker paid \$155.76, including 10 percent state and federal taxes, for diesel fuel. What was the price per gallon for the fuel if the taxes are excluded?
- (1) The trucker paid \$0.118 per gallon in state and federal taxes on the fuel last Tuesday.
 - (2) The trucker purchased 120 gallons of the fuel last Tuesday.
14. Is x less than y ?
- (1) $x - y + 1 < 0$
 - (2) $x - y - 1 < 0$
15. What is the value of x ?
- (1) When x is multiplied by 8, the result is between 50 and 60.
 - (2) When x is doubled, the result is between 10 and 15.
16. Car X and car Y ran a 500-kilometer race, What was the average speed of car X?
- (1) Car X completed the race in 6 hours and 40 minutes.
 - (2) Car Y, at an average speed of 100 kilometers per hour, completed the race 1 hour and 40 minutes before car X crossed the finish line.

17. If the successive tick marks shown on the number line above are equally spaced and if x and y are the numbers designating the end points of intervals as shown, what is the value of y ?

(1) $x = \frac{1}{2}$

(2) $y - x = \frac{2}{3}$

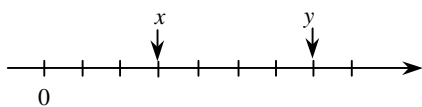


图 6.7

参考答案

1~5 C E A E

6~10 B A E B D

11~15 A E D A E

16~17 D D

第7章 疑难杂症

除了前面几章谈到的粗心问题、思维陷阱、知识点的遗忘及理解错误外，考生常常反映的另外一个问题是自己根本无法下手，这不属于以上谈过的任何一个原因，但是就是不会做。

很多考生存在的还有一个问题是反应太慢，缺乏解题的技巧，无法在短时间内完成考试。

本章即是针对以上两个问题，将考试中无法下手的问题分类总结，并将作者多年来在 GMAT 数学考试和辅导中常用的方法奉献出来，以飨广大读者。

7.1 无法转换为数学问题(1) ——整除问题

日常生活中的很多问题都可以转化为整除问题。整除问题是考查的重点之一，并且会以两种方式——数学背景和日常生活背景给出。

例 1: In the figure below, how many of the points on line segment PQ have coordinates that are both integers?

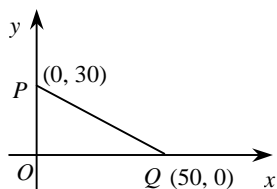


图 7.1

- (A) 5 (B) 8
(C) 10 (D) 11 (E) 20

翻译: 在下面的图形中，线段 PQ 上有多少点的坐标都是整数？

解答: 上面的线段方程为 $y = -\frac{3}{5}x + 30$

($0 \leq x \leq 50$)，则若坐标都是整数， x 坐标都应被 5 整除，在 0 至 50 之间可以被 5 整除的数有 11 个。

所以(D) is correct.

错误分析: 很多考生在本题目目前都不知所措，若能写出直线方程，并且将本题转化为一道整除问题，本题即迎刃而解。

提示: 对于坐标几何中的图形，一般要将其转化为方程问题，实现“形”对“数”的转换，方可进一步求解。

例 2: What is the tens digit of positive integer x ?

(1) x divided by 100 has a remainder of 30.

(2) x divided by 110 has a remainder of 30.

翻译: 正整数 x 的十位数的数字是多少？

(1) x 被 100 除时余数为 30

(2) x 被 110 除时余数为 30

解答: 由(1)可得 $x = 100n + 30$ ，无论 n 取何值 x 的十位数的数字都是 3，所以(1)单独回答了问题；由(2)可得 $x = 110n + 30$ ，则 x 的十位数的数字可能为多个，(2)单独不能回答问题。

所以(A) is correct.

错误分析: 很多人觉得本题的条件和问题没有丝毫联系, 因为他们根本没有将条件“ x divided by 100 has a remainder of 30”转化为数学的语言。

提示: 对于“A除以B余C”的问题, 首先应转化为 $A=mB+C$, m 为整数。

例 3: If the two-digit integers M and N are positive and have the same digits, but in reverse order, which of the following CANNOT be the sum of M and N ?

- (A) 181
- (B) 165
- (C) 121
- (D) 99
- (E) 44

翻译: 假如两位数的整数 M 和 N 是正整数并且有相同的数字, 但是顺序相反。下列哪一个不能是 M 和 N 的和?

解答: 设 M 的个位为 x , 十位为 y , 则 $M = 10y + x$, 则 $N = 10x + y$, 则 $M + N = 11(x + y)$, 则需从 5 个选项中找到不是 11 的倍数的数, 显然(A)中的 181 不是 11 的倍数。

所以(A)is correct.

提示: 本题考查的是组成一个数的数字(digit)与这个数的数值(number)之间的关系。

例 4: If M and N are positive integers that have remainders of 1 and 3, respectively, when divided by 6, which of the following could NOT be a possible value of $M + N$?

- (A) 86
- (B) 52
- (C) 34
- (D) 28

(E) 10

翻译: 假如 M 和 N 是正整数且被 6 除时余数分别为 1 和 3, 下列哪一个不能是 $M + N$ 的可能值?

解答: 设 $M = 6m + 1$, $N = 6n + 3$, 则

$$M + N = 6(m + n) + 4$$

5 个选项中只有(A)86 不能表达为上面的模式。

所以(A)is correct.

提示: 对于此类问题, 首先应转化为 $A=mB + C$, 其中 A 为被除数, B 为除数, C 为余数, m 为整数。

例 5: If n is an integer, which of the following CANNOT be a factor of $3n+4$?

- (A) 4
- (B) 5
- (C) 6
- (D) 7
- (E) 8

翻译: 假如 n 是一个整数, 下列哪一个不能是 $3n+4$ 的因子?

解答: 方法 1: 设 m 为 $3n+4$ 的因子, 则有:

$$3n+4=km \quad n=\frac{km}{3}-\frac{4}{3}$$

其中 k 为整数, n 为整数, 故 $\frac{km}{3}$ 一定

不是整数, m 不应该是 3 的倍数。

所以(C)is correct.

方法 2: 可采用排除法, (A)当 $n=4$ 时, 4 可以为 $3n+4$ 的因子; (B)当 $n=7$ 时, 5 也可以为 $3n+4$ 的因子; (C)无论 n 取何值, $3n+4$ 都不可能为 6 的因子, 而 (D)中 n 取 1 时可以被 7 整除; (E)中 n 取 4 时可以被 8 整除, 所以(C)is correct.

提示: 方法 1 是整除问题的基本方法, 由于 GMAT 数学全部以选择题的形式给出, 考试时考生也可以采用排除法, 特殊值

法等特殊解法。

例 6: A necklace is made by stringing N individual beads together in the repeating pattern red bead, green bead, white bead, blue bead, and yellow bead. If the necklace design begins with a red bead and ends with a white bead, then N could equal?

- (A) 16
(B) 32
(C) 41
(D) 54
(E) 68

翻译: 一个项链由 N 个不同的珠子制成, 珠子为以红色、绿色、白色、蓝色和黄色的重复形态。假如项链的设计以红色珠子开始且以白色珠子结束, 则 N 的值可以是多少?

解答: 白色为第三个珠子, 所以 N 的值应满足: $N = 5n + 3$

$68 = 5 \times 13 + 3$ 满足上面的式子。

所以(E) is correct。

提示: 本题的关键是考生能够将题中的描述转化为数学问题: $N = 5n + 3$ 。其中 n 为整数。

7.2 无法转换为数学问题(2) ——排列组合问题

例 1: Pat will walk from intersection X to intersection Y along a route that is confined to the square grid of four streets and three avenues shown in the map above. How many routes from X to Y can Pat take that have the minimum possible length?

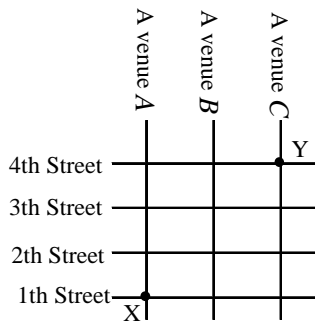


图 7.2

- (A) Six
(B) Eight
(C) Ten
(D) Fourteen
(E) Sixteen

翻译: P 将从相交点 X 徒步到相交点 Y, 沿着如图 7.2 所示的 4 条横向街道和 3 条纵向街道所构成的路线, 且街道与街道相交成正方形, 从 X 到 Y 最短可能长度的路线数目是多少?

解答: 从 X 到 Y 的最短长度为 5 个(如图 7.2 所示)正方形的边长, 但路线数目该怎样计算呢? 有些考生通过硬数将答案数出, 这样不但可能发生错误, 当路线进一步复杂时, 这种方法就不适用了, 所以需体会本题正确的计算方法(注意: 当考生实在不会用排列组合计算时, 硬将线数数出且采用排除法不失为一个好的解决方法):

要想使路线最短, 经过 2、3、4 的顺序一定是固定的, 而经过 B、C 的顺序也肯定固定, 否则不可能路线最短, 且必然经过 2、3、4、B、C 这 5 条直线, 所以本题转化为 2、3、4、B、C 这 5 个元素的排列, 且满足顺序分别为 2、3、4 和 B、C, 则总的排列可能通过两种方法得到:

① 5 个位置中选出两个位置给 B、C

且满足 B 在 C 前, 则为 C_5^2 , 剩下 3 个位置也必然按顺序为 2、3、4 这种惟一可能, 所以 $C_5^2 = 10$ 。

② 5 个位置中选出 3 个位置给 2, 3, 4 且必须满足 2 在 3 之前, 3 在 4 之前, 则为 C_5^3 , 剩下两个位置也必然按顺序 B 在 C 的前面这种惟一可能, 所以 C_5^3 。

所以(C)is correct。

错误分析: 很多考生没有将本题与排列组合问题联系起来, 根本无从下手。

例 2: Ben and Ann are among 7 contestants from which 4 semifinalists are to be selected. Of the different possible selections, how many contain neither Ben nor Ann?

- (A) 5
- (B) 6
- (C) 7
- (D) 14
- (E) 21

翻译: Ben 和 Ann 在 7 名竞争者中, 从这 7 人中选出 4 名半决赛选手。不包括 B 和 A 的选择一共有多少种?

解答: 不包括 Ben 和 Ann 则剩下 5 人, 本题意为从 5 人中选出 4 人的可能性有多少种, 即 $C_5^4 = 5$ 种。

所以(A)is correct。

错误分析: 其实本题就是“5 人中选出 4 人的可能性有多少种”, 如果这样去问的话, 每个 GMAT 的考生都不会有问题。有了 Ben 和 Ann 的干扰, 很多考生都没有答对本题。

提示: GMAT 在考试中会设置许多干扰元素, 在选项中也会有不少干扰。考生应该学会排除干扰, 抓住问题的核心。

7.3 无法转换为数学问题(3) ——极值问题

极值问题也是 GMAT 考查的重点之一, 很多考生都很头疼。考生在解答时应遵循以下 3 个步骤:

- (1) 读题, 将题目中的文字描述转换为数学问题, 明确几个未知数之间应该满足是等式还是不等式的关系。
- (2) 列式(等式或者不等式), 注意其他限制条件如必须是整数等。
- (3) 将等式或者不等式进行变形, 求解。

例 1: Kim bought a total of \$2.65 worth of postage stamps in four denominations. If she bought an equal number of 5-cent and 25-cent stamps and twice as many 10-cent stamps as 5-cent stamps, what is the least number of 1-cent stamps she could have bought?

- (A) 5
- (B) 10
- (C) 15
- (D) 20
- (E) 25

翻译: K 共购买了价值 2.65 美元的 4 种面值的邮票。假如她购买了相同个数的 5 美分和 25 美分的邮票, 并且 10 美分邮票的个数是 5 美分邮票的 2 倍, 她所购买的 1 美分邮票的最少数目是多少?

解答: 设 K 购买了 x 张 5 美分的邮票并且购买了 y 张 1 美分的邮票, 则购 10 美分的邮票有 $2x$ 张, 25 美分的邮票有 x 张得到如下方程:

$$5x + 25x + 10 \times 2x + y = 265$$

$$y = 265 - 50x$$

若使 y 最小则 x 取最大值为 5, 得到

$$y = 15,$$

所以(C) is correct.

提示: 极值问题的首先要转化为数学问题, 根据题意列等式或者不等式。

例 2: Erica has \$460 in 5- and 10-dollar bills only. If she has fewer 10- than 5-dollar bills, what is the least possible number of 5-dollar bills she could have?

- (A) 32 (B) 30
(C) 29 (D) 28
(E) 27

翻译: E 仅有 5 美元和 10 美元的钞票共 460 美元。假如她手中的 10 美元钞票比 5 美元的钞票少, 她可能有的 5 美元钞票的数目最少是多少?

解答: 设 E 的 m 张 5 美元的钞票, n 张 10 美元的钞票, 且 $m > n$, 则

$$5m + 10n = 460$$

$$m + 2n = 92$$

观察得 $m = 32, n = 30$

则 5 美元钞票的最小值为 32。

所以(A) is correct.

错误分析: 注意本题的限制条件“10 美元钞票比 5 美元的钞票少”。

例 3: In the diagram below, points A, B, C, D, and E represent the five teams in a certain league in which each team must play each of the other teams exactly once. The segments connecting pairs of points indicate that the two corresponding teams have already played their game. The arrows on the segments point to the teams that lost; the lack of an arrow on a segment indicates that the game ended in a tie. After all games have been played, which of the following could NOT be the

percent of games played that ended in a tie?

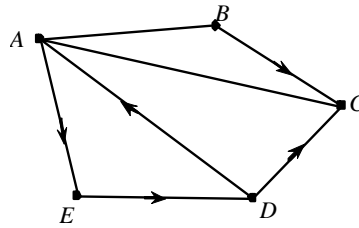


图 7.3

- (A) 10%
(B) 20%
(C) 30%
(D) 40%
(E) 50%

翻译: 在图 7.3 中, 点 A、B、C、D 和 E 代表某一联盟的 5 支球队, 其中每支队伍与其他队伍比赛一次。连接每对点之间的线段指出两支相对应的队伍已经比赛了。线段上箭头所对的点指失利的队伍; 没有箭头的线段指出比赛以平局结束。当所有的比赛都结束后, 下列哪一项不可能是以平局结束比赛所占的百分比?

解答: 5 个队伍共进行 C_5^2 场比赛即 10 场比赛, 如图 7.3 所示, 我们可以看到已进行了 7 场比赛, 其中有 2 场比赛是平局, 则平局已经占了总比赛数目至少 20% 的比例, 10% 不可能是平局所占的百分比。

所以(A) is correct.

提示: 本题有两个关键, 其一是对图表的理解, 其二是对极值问题的理解。如果一个过程分为几个阶段且各个阶段相互独立, 整个过程的最大值等于各个阶段最大值之和; 最小值亦然。

本题可以看成是 2 个阶段。第一阶段是已经进行的比赛, 其平局的最大值和最小值

都是2, 因为比赛已经结束; 第二阶段是将要进行的比赛, 平局的最大值是3(全部是平局), 最小值是0(全部分出胜负)。因此所有的比赛, 平局的最大值是5(占50%), 最小值是2(占20%)。

7.4 无法转换为数学问题(4) ——其他问题

例 1: The figure below represents the floor plan of an art gallery that has a lobby and 18 rooms. If Lisa goes from the lobby into room A at the same time that Paul goes from the lobby into room R and each goes through all of the rooms in succession, entering by one door and exiting by the other, which room will they be in at the same time?

- (1) Lisa spends $2x$ minutes in each room and Paul spends $3x$ minutes in each room.
- (2) Lisa spends 10 minutes less time in each room than Paul does.

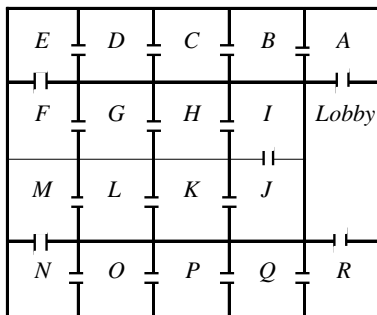


图 7.4

翻译: 图 7.4 是有一个门厅和 18 个房间的艺术馆的布局方案。假如 Lisa 从门厅进入房间 A 的同时 Paul 从门厅进入房间 R, 并且每人都连续经过所有的房间通过从一个门进入而从另一个门出来, 他们将同时在哪一个房间?

(1) Lisa 在每个房间中呆 $2x$ 分钟, Paul 在每个房间中呆 $3x$ 分钟

(2) Lisa 比 Paul 在每个房间中少呆 10 分钟

解答: 如上图, 每个人经过房间的次序是一定的, 由(1)设 Lisa 和 Paul 在 Lisa 所参观的第 n 个房间相遇, 则得到如下方程: $2xn = 3x(18 - n)$

两边同时消掉 x 解出 n , 所以(1)单独可以回答问题; (2)设 Lisa 在每个房间呆 x 分钟, 则 Paul 在每个房间呆 $x + 10$ 分钟, 同时设 Lisa 和 Paul 在 Lisa 所参观的第 n 个房间相遇, 得到如下方程:

$$x \times n = (18 - n)(x + 10)$$

因为存在两个未知数, 所以无法求得 n , (2)单独无法回答问题。

所以(A)is correct。

提示: 很多人在本题前无法下手, 浪费了很多时间, 最后还是选择了猜答案。由于本题的门的位置一定, 每个人经过房间的次序是也就确定了, 即 Lisa 以 A B C D... R 的顺序, Paul 以相反的顺序, 也就是运动问题中的相向运动。

例 2: According to the incomplete table below, if each of the 6 teams in the league played each of the other teams exactly twice and there were no ties, how many games did team X win? (Only 2 teams play in a game.)

表 7.1 LEAGUE RESULTS

Team	Number Gores Wan
A	4
B	7
C	9
D	2
E	2
X	

- (A) 4
(B) 5
(C) 6
(D) 8
(E) 10

翻译: 根据上面不完全的图 7.5 所示, 假如联盟中 6 支队伍中的每一支与另 5 支队伍的每一支共比赛两次且没有平局, X 赢了多少场比赛(每一场比赛中仅有两支队伍比赛)?

解答: 6 支队伍中任两支比赛两次, 则共比赛了 $2C_6^2 = 30$ 场

则共 30 次获胜, 那么 x 获胜的次数为:

$$30 - (4 + 7 + 9 + 2 + 2) = 6$$

所以(C)is correct.

错误分析: 由于没有平局, 所有球队赢的总场次等于比赛总的场次, 因此本题可转化为一个排列组合问题。想通了这一点, 本题应该没有多少难度。

例 3: In a certain game, a large container is filled with red, yellow, green and blue beads worth, respectively, 7, 5, 3, and 2 points each. A number of beads are then removed from the container. If the product of the point values of the removed beads is 147,000, how many red beads were removed?

- (A) 5 (B) 4
(C) 3 (D) 2
(E) 0

翻译: 假如在某一比赛中, 一个大容器中有红色、黄色、绿色和蓝色珠子, 其分值分别为 7、5、3 和 2 点。若将一些珠子从容器中取出, 假如多出的珠子点的乘积为 147000 点, 有多少红色的珠子被

移出?

解答: 本题意为取出的珠子点数的乘积为 147000, 则将 147000 拆写成质因子连乘积的形式为:

$$147000 = 2^3 \times 3^1 \times 5^3 \times 7^2$$

由 $r = 7, y = 5, g = 3, b = 2$, 则红色珠子共两个被取出。

所以(D)is correct.

提示: 本题其实和容器, 珠子等都没有太多关系, 关键是求 147000 的质因子。

例 4: What is the units digit of $(13)^4 (17)^2 (29)^3$?

- (A) 9 (B) 7
(C) 5 (D) 3
(E) 1

翻译: $(13)^4 (17)^2 (29)^3$ 的个位数字是多少?

解答: $(13)^4$ 的个位数字为 1

$(17)^2$ 的个位数字为 9

$(29)^3$ 的个位数字为 9

则 $(13)^4 (17)^2 (29)^3$ 的个位数字为 1。

所以(E)is correct.

错误分析: 这是一道非常“唬人”的题目, 有的考生甚至去求 $(13)^4 (17)^2 (29)^3$ 的值。其实, 若每个乘数的个位数字已知, 其乘积的个位数字也就已知了, 与乘数的其他位数字没有关系。

例 5: A cashier mentally reversed the digits of one customer's correct amount of change and thus gave the customer an incorrect amount of change. If the cash register contained 45 cents more than it should have as a result of this error, which of the following could have been the correct amount of change in cents?

- (A) 14 (B) 45
- (C) 54 (D) 65
- (E) 83

翻译: 一个营业员糊涂地把找给一个顾客的零钱的数字弄颠倒了, 以至于给了顾客不正确的钱, 假如因为这个错误而发现多了 45 美分, 下列哪一个可能是以美分计的零钱的正确数目?

解答: 两个数字颠倒了以至于多了 45 美分, 则设正确零钱的十位数为 x , 个位数为 y , 则

$$(10x + y) - (10y + x) = 45$$

$$x - y = 5$$

则十位数减个位数的值应为 5,

所以(E)is correct.

提示: 本题应该去设所求值的十位数和个位数, 而不是正确零钱的数目, 否则无法求解。

例 6: How many integers n greater than 10 and less than 100 are there such that, if the digits of n are reversed, the resulting integer is $n + 9$?

- (A) 5 (B) 6
- (C) 7 (D) 8
- (E) 9

翻译: 有多少整数 n 大于 10 且小于 100, 并且假如 n 的数字被颠倒, 所得的数字为 $n+9$?

解答: 设 n 的个位数字为 x 。十位数字为 y , 则

$$n = 10y + x$$

则可以得到:

$$10y + x + 9 = 10x + y$$

$$x - y = 1$$

即个位数字比十位数字大 1。满足此特点的数字有 12、23、34、45、56、67、78、89 共 8 个数字, 所以(D)is correct。

例 7: In the addition problem below, the number \square must be

$$\begin{array}{r} 734 \\ 5\square 8 \\ + 9\square 2 \\ \hline 2,2\square 4 \end{array}$$

- (A) 5
- (B) 6
- (C) 7
- (D) 8
- (E) 9

翻译: 在下面的加法算式中, 数 \square 一定是多少?

解答: 由加法的原则, 则得到如下算式:

$$3 + \square + \square + = 10 + \square$$

$$\square = 6$$

所以(B)is correct.

提示: 注意进位。

例 8: In the figure below, two searchlights S_1 and S_2 are located 10,000 feet apart, each covers an area of radius 10,000 feet, and each is located 8,000 feet from the railroad track. To the nearest 1,000 feet, what is the total length x of track spanned by the searchlights?

- (A) 24,000
- (B) 22,000
- (C) 20,000
- (D) 16,000
- (E) 12,000

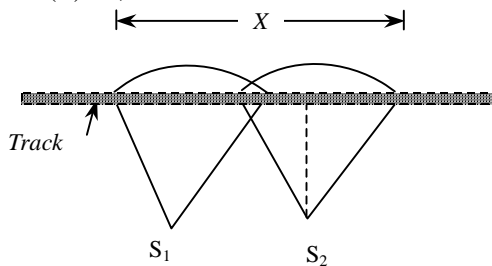


图 7.5

翻译: 如图 7.5 所示, 两个探照灯 S_1 和 S_2 相距 10000 英尺, 每个覆盖面积的半径为 10000 英尺, 并且每个探照灯距铁轨的距离为 8000 英尺。四舍五入到 1000 英尺, 探照灯所覆盖的铁轨的总长度 x 是多少?

解答: 由 S_1 S_2 向铁道做垂线, 则两条垂线之间的距离为 10000 英尺, 若求 x 的长度, 则需求出两边的两段直角边的长度。根据勾股定理得:

$$[(1000)^2 - (3000)^2]^{1/2} = 6000 \text{ 英尺}$$

所以 x 的长度为:

$$6000 \times 2 + 10000 = 22000 \text{ 英尺}$$

所以(B)is correct。

错误分析: 有很多人忽略了两个探照灯覆盖的铁轨相互交叉, 于是 $x = 6000 \times 4 = 24000$, 从而错误选择了选项(A)。

例 9: A printer numbered consecutively the pages of a book, beginning with 1 on the first page. In numbering the pages, he printed a total of 189 digits. How many pages are there in the book?

- (A) 98
- (B) 99
- (C) 100
- (D) 101
- (E) 189

翻译: 一个打印员为一本书的书页进行编号, 从第一页 1 开始, 在计算页码时, 他共打印了 189 个数字。问这本书一共有多少页?

解答: digit n . 数字, 指一个阿拉伯数字, 诸如 1、2、3 等。若 22、35、78 之类的两位数, 表示为 “two-digit number”, 诸如 125、767 等 3 位数, 表示为

“three-digit number”, 则该打印员标号共标了 189 个数字, 除去 1 位数的 9 个数字, 则两位数共 90 个, 共 99 页, 所以与 100 比较, $99 < 100$ 。

所以(B)is correct。

错误分析: 有很多考生没有搞清楚 digit 与 number 的区别, 因此无法下手。

例 10: Each of the following numbers has two digits blotted out, Which of the numbers could be the number of hours in x days, where x is an integer?

- (A) 25■, ■06
- (B) 50■, ■26
- (C) 56■, ■02
- (D) 62■, ■50
- (E) 65■, ■20

翻译: 下列数中的每一个数都有 2 个数字被涂掉, 哪一个表示 x 天的小时数(x 是一个整数)?

解答: 因为 4 为 24 的一个因子, 被 24 整除, 一定要被 4 整除。例如把(A)选项拆开为 $25 \blacksquare \blacksquare 00 + 06$, 则前一项必然能被 4 整除且无余数, 那么这个数要想被 4 整除, 必须十位和各位可以被 4 整除, 显然(A)、(B)、(C)、(D)均不满足这一条件, 惟一可能的是(E)。

所以(E)is correct。

提示: 对于数字做合理变形, 是解决本类问题的关键。

例 11: The participants in a race consisted of 3 teams with 3 runners on each team. A team was awarded $6 - n$ points if one of its runners finished in n th place, where $1 \leq n \leq 5$. If all of the runners finished the race and if there were no ties, was each

team awarded at least one point?

- (1) No team was awarded more than a total of 6 points.
 (2) No pair of teammates finished in consecutive places among the top five places.

翻译: 一个比赛的参与者为 3 组, 每组 3 名赛跑者。假如一组一名赛跑者得了第 n 名, $1 \leq n \leq 5$, 该组被给予 $6 - n$ 分, 假如所有的赛跑者都完成了比赛并且没有平局, 每组至少得了 1 分吗?

- (1) 没有一个组得分超过 6 分。
 (2) 在前 5 名中没有同组成员获得连续名次。

解答: 由题意, 前 5 名分别得 5、4、3、2、1 分。(1)中说明没有一个组得分超过 6 分, 则 3 组中每组都至少得了 3 分, 所以(1)回答了上面的问题; (2)中说明前 5 名中, 无同组成员获得连续名次。可能某一组成员为 1、3、5 名, 另一组成员为 2、4 名, 则无法回答上面的问题。所以(A)is correct。

7.5 GMAT数学常用方法

7.5.1 数形结合

学会数形结合, 特别是在做几何、集合或概率方面的题时, 将数转化为形是解决很多问题的关键, 常常能够帮助考生准确迅速地解题。

例 1: If 75 percent of a class answered the first question on a certain test correctly, 55 percent answered the second question on the test correctly, and 20 percent answered neither of the questions correctly, what percent answered both

correctly?

- (A) 10%
 (B) 20%
 (C) 30%
 (D) 50%
 (E) 65%

翻译: 假如在某一测试中某班级有 75% 的人正确回答了第一个问题, 55% 的人正确回答了第二个问题, 20% 的人没有答对任一道题, 两道题都回答正确的百分比是多少?

解答: 因为有 20% 的人任一道题都未答对, 所以两道题中至少答对一道题的比率为 80%, 则两道题都答对的百分比为: $55\% + 75\% - 80\% = 50\%$ 所以(D)is correct。

提示: 对于集合问题, 如果考生没有完全理解或者记住公式, 可以通过图形来帮助推理。

例 2: If X and Y are sets of integers, $X \triangle Y$ denotes the set of integers that belong to set X or set Y , but not both. If X consists of 10 integers, Y consists of 18 integers, and 6 of the integers are in both X and Y , then $X \triangle Y$ consists of how many integers?

- (A) 6
 (B) 16
 (C) 22
 (D) 30
 (E) 174

翻译: 假如 X 和 Y 是整数的集合, $X \triangle Y$ 定义为属于集合 X 或属于集合 Y , 但不是两者共同拥有的整数集合。假如 X 中包括 10 个整数, Y 中包括 18 个整数, 并且有 6 个整数同时在 X 和 Y , 那么 X

$\triangle Y$ 包括多少个整数?

解答: $X\triangle Y$ 中的整数数目为:

$$10+18-6-6=16$$

所以(B) is correct.

提示: 本题和例 1 很相似, 考生也可以通过图形来帮助推理。

Example 3 and 4 refer to the following information:

In a marketing survey for products A, B, and C, 1,000 people were asked which of the products, if any, they use. The three circular regions in the diagram below represent the numbers of people who use products A, B, and C, according to the survey results. Of the people surveyed, a total of 400 use A, a total of 400 use B, and a total of 450 use C.

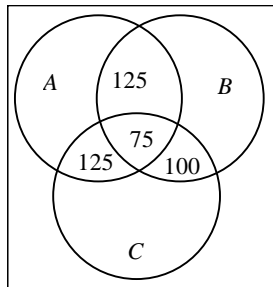


图 7.6

翻译: 例 3 和例 4 根据下面的信息:

在一个对于产品 A, B, C 的市场营销的调查中, 1000 人被问及他们使用哪一种产品。如图 7.6 中的 3 个圆形区域代表使用产品 A, B, C 的人数, 根据调查的结果, 在被调查的人中, 共有 400 人使用 A, 400 人使用 B, 450 人使用 C。

例 3: How many of the people surveyed use exactly one of the products?

(A) 75

(B) 100

(C) 150

(D) 250

(E) 325

翻译: 调查中有多少人只使用一种产品?

解答: 如图 7.7 所示, 仅使用一种产品的人一定是不含数字的区域, 单独使用 A 产品有的人:

$$400 - 125 - 125 - 75 = 75$$

单独使用 B 产品的人有:

$$400 - 125 - 75 - 100 = 100$$

单独使用 C 产品的人有:

$$450 - 125 - 75 - 100 = 150$$

所以仅使用一种产品的人有:

$$75 + 100 + 150 = 325$$

所以(E) is correct.

错误分析: 注意本题要求的是“只使用一种产品的人”。

例 4: What percent of the people surveyed use product A or product B or both, but not product C?

(A) 12.5%

(B) 17.5%

(C) 30%

(D) 40%

(E) 60%

翻译: 调查中有百分之多少的人或者使用 A, 或者使用 B, 或者两者同时使用, 但不使用 C?

解答: 使用 A 但不使用 C 的人有:

$$400 - 125 - 75 = 200$$

使用 B 但不使用 C 的人有:

$$400 - 75 - 100 = 225$$

使用 A 或 B 或两者同时使用的人有:

$$200 + 225 - 125 = 300$$

所以所占百分比为:

$$300/1000 = 30\%$$

(C) is correct.

提示: 实现数对形的转化, 是复杂集合问题的最优解法。

例 5: All trainees in a certain aviator training program must take both a written test and a flight test. If 70 percent of the trainees passed the written test, and 80 percent of the trainees passed the flight test, what percent of the trainees passed both tests?

- (1) 10 percent of the trainees did not pass either test.
 (2) 20 percent of the trainees passed only the flight test.

翻译: 某一飞行员培训项目的所有受训者, 必须参加写作测试和飞行测试。假如 70% 的受训者通过了写作测试, 80% 的受训者通过了飞行测试。同时通过两次测试的受训者的百分比是多少?

- (1) 10% 的受训者未通过任一测试。
 (2) 20% 的受训者仅仅通过飞行测试。

解答: 由(1)10%的人未通过任一测试, 得到 90%的人至少通过 1 个测试, 则通过两个测试的人可以通过 $(70\%+80\%)-90\%$ 得到, 所以(1)可以单独回答问题; (2)20%的人仅通过飞行测试说明了通过两个测试的人有 $80\%-20\%=60\%$, (2) 单独回答了上面的问题

所以(D) is correct.

提示: 解题时考生不要只依赖于大脑的思考, 如果能够动手作图, 实现数对形的转化, 本题即迎刃而解。

例 6: A ladder 25 feet long is leaning against a wall that is perpendicular to level ground.

The bottom of the ladder is 7 feet from the base of the wall. If the top of the ladder slips down 4 feet, how many feet will the bottom of the ladder slip?

- (A) 4
 (B) 5
 (C) 8
 (D) 9
 (E) 15

翻译: 一个 25 英尺的梯子被斜靠在一堵与地面垂直的墙上, 梯子底部距墙的底部有 7 英尺。假如梯子顶部向下滑 4 英尺, 梯子的底部将滑多少英尺?

解答: 在未滑动之前, 梯子的垂直高度为:

$$\sqrt{25^2 - 7^2} = 24 \text{ 英尺}$$

顶部向下滑 4 英尺, 则梯子的垂直高度为 20 英尺, 则梯子距墙的底部为:

$$\sqrt{25^2 - 20^2} = 15 \text{ 英尺}$$

$$15 - 7 = 8 \text{ 英尺}$$

所以(C) is correct.

错误分析: 很多人没有答对本题, 因为他们根本没有明白题目描述的是什么样的过程。如果能够做出如图 7.7 所示的图形, 本题就没有太多的难度了。

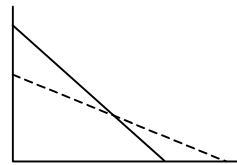


图 7.8

7.5.2 极限法

本章 7.3 节中, 我们谈到了极值问题, 本小节中再介绍极值法。极值问题是考查的重点之一, 而极值法是考生应该掌握的非常重要的方法之一。

极值问题的基本思想包括:

1. 如果一个过程分为几个阶段且各个

阶段相互独立, 整个过程的最大值等于各个阶段最大值之和; 最小值亦然。

2. n 个元素和的最大值, 等于各个元素最大值之和; 最小值亦然。

例: There are 4 card-processing machines in an office. The fastest of these machines processes x cards in 7 hours and the slowest processes x cards in 8 hours. Which of the following could NOT be the average time per machine for each of the 4 machines to process x cards?

- (A) 7.2
- (B) 7.3
- (C) 7.5
- (D) 7.6
- (E) 7.7

翻译: 在一个办公室中有 4 个卡片处理机器, 这些机器中, 最快的可在 7 小时内处理 x 张卡片, 最慢的可以 8 小时内处理 x 张卡片。下列哪一个不可能是 4 台机器处理 x 张卡片的平均时间?

解答: 本题已知最快的处理 x 张卡片的时间为 7 小时, 而最慢的为 8 个小时, 其余 3 台机器处理时间未知, 但必然在 7 至 8 小时内, 则 5 台机器的平均值的最小值为:

$$(7+7+7+8)/4=7.25 \text{ 小时}$$

5 台机器处理 x 张卡片的最大平均值为:

$$(7+8+8+8)/4=7.75 \text{ 小时}$$

由平均值的范围得 7.2 不是平均值,

所以(A)is correct。

提示: 本题平均值的范围应该属于最大值 7.75 和最小值 7.25, 并且不包括边界值。是否包括边界值, 也是极值问题考查的发现之一。

7.5.3 代入法

GMAT 数学考试可能有一些新型的问题, 考生从没有见过也无法下手。此时考生不妨采用代入法, 将答案代入原题, 排除或者确定答案。

例 1: When 66, 90, 150 are divided by the positive integer n respectively, they have the same remainder, which of the following could be the largest value of n ?

- (A) 3
- (B) 4
- (C) 8
- (D) 12
- (E) 16

翻译: 66、90、150 分别除以正整数 n 时, 余数相同, n 的最大值可能是下面哪一个?

解答: 方法 1: 如果不知道如何求解, 可以用代入排除法比较, 求出最大的一个数。因为要求 n 的最大值, 所以可以首先代入最大的一个选项, 若不成立依次减小。

$$66=12 \times 5+6$$

$$90=12 \times 7+6$$

$$150=12 \times 12+6$$

所以(D)is correct。

方法 2: 这类题目通常利用同余概念来求解, 将三个数中任意两个数相减, 其差就能够被 n 整除。

$$90-66=24, \text{ 24 能够被 } n \text{ 整除;}$$

$$150-90=60, \text{ 60 也能够被 } n \text{ 整除;}$$

因为 24 和 60 的最大公约数是 12。

所以(D)is correct。

提示: 对于某些极值问题和一些新型问题, 考生若无法下手, 不妨采用代入排除法, 将答案代入原题。

若求最大值, 首先应该代入最大的一个选项, 若不成立依次减小; 若求最小值, 首先应该代入最小的一个选项, 若不成立依次增加。

7.5.4 列举法

列举法的基本思想是列举所有可能的情况, 然后进行解答。GMAT 数学中有些问题只有通过列举法才可能求解, 也有一些问题列举法不是必须却是最优的。

例 1: The table below shows the cost, in dollars, of traveling to and from cities A, B, C, D, E, and F. A sales representative wants to leave from A, travel to C, E and F, and return to A. If the first city that the sales representative travels to must be E, what is the minimum possible cost for the entire trip?

	<i>To</i>						
		A	B	C	D	E	F
<i>From</i>	A		3	3	2	7	3
	B	3		3	4	5	5
	C	3	3		1	2	4
	D	2	4	1		5	5
	E	7	3	2	5		6
	F	3	5	4	5	6	

图 7.8

- (A) \$13
- (B) \$14
- (C) \$16
- (D) \$18
- (E) \$20

翻译: 如图 7.8 所示, 从 A、B、C、D、E 和 F 中的任一城市到其他城市的旅行

花费, 以美元计。一个销售代表想离开 A, 旅行到 C、E、F, 然后回到 A, 假如销售代表第一个到达的城市一定是 E, 整个旅途的最低花费是多少?

解答: 由 A 到 E 花费 7 美元, 至此该销售代表有两种可选择的道路, 其 1 为:

$$A \rightarrow E \rightarrow C \rightarrow F \rightarrow A$$

其花费为:

$$7 + 2 + 4 + 3 = 16 \text{ 美元}$$

另一道路为:

$$A \rightarrow E \rightarrow F \rightarrow C \rightarrow A$$

其花费为:

$$7 + 6 + 4 + 3 = 20 \text{ 美元}$$

综上所述道路 1 的花费最低为 16 美元。

所以(C)is correct.

提示: 本题是一个简单的极值问题, 只有两种可能的情况, 考生只要列举出这两种选择进行比较即可。

例 2: If n is a prime number greater than 3, what is the remainder when n^2 is divided by 12?

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) 5

翻译: 假如 n 是一个大于 3 的质数, 当 n^2 被 12 整除时的余数是多少?

解答: 任取一个大于 3 的质数尝试, 发现任一 n^2 被 12 整除时, 余数总为 1, 例如 $5^2 = 25$, $25 \div 12$ 得到商为 2, 余数为 1。

所以(B)is correct.

提示: 考生应该注意到本题的答案是惟一的, 即无论考生选取哪一个质数, 答案都应该是相等的。本题属于特殊的列举法, 考生只需要任意选择“一个大于 3

的质数”。

例 3: The inside dimensions of a rectangular wooden box are 6 inches by 8 inches by 10 inches. A cylindrical canister is to be placed inside the box so that it stands upright when the closed box rests on one of its six faces. Of all such canisters that could be used, what is the radius, in inches, of the one that has maximum volume?

- (A) 3
(B) 4
(C) 5
(D) 6
(E) 8

翻译: 长方体木盒的内部的三维分别为 6 英寸、8 英寸和 10 英寸，一个圆柱形的罐子被放置在盒子中，以至于当关闭的盒子以其六个面的任何一个面为底时都保持直立。在所有可以被使用的这种罐子中，有最大体积的罐子的半径是多少？

解答: 因为无论该长方体以哪一个面为底时，圆柱体都保持直立，则该圆柱体一定内切于该长方体，什么样的情况下该圆柱体体积最大呢？因为圆柱体的体积为： $\pi r^2 \times h$

当圆柱体以 6 inches by 8 inches 为底面时，体积为： 90π

当圆柱体以 6 inches by 10 inches 为底面时，体积为： 72π

当圆柱体以 8 inches by 10 inches 为底面时，体积为： 96π

所以半径为 4 时，体积最大，

所以(B)is correct。

错误分析: 本题有 3 种可能情况，考生只要逐个列举进行比较即可。很多考生由于

没有读懂题而无法下手。

例 4: In the addition problem above, each of the symbols \square , \triangle , and \star represents a positive digit. If $\square < \triangle$, what is the value of \triangle ?

$$\begin{array}{r} \square \\ + \triangle \\ \hline \star \end{array}$$

图 7.9

- (1) $\star=4$
(2) $\square=1$

翻译: 在左面的加法问题中， \square 、 \triangle 、 \star 这些符号代表一个正的数字，假如 $\square < \triangle$ ，那么 \triangle 的值是多少？

解答: 如图 7.9 所示的加法表明 $\square + \triangle = \star$ ，且这 3 个符号都表示一个正的数字，注意 digit 这个单词指一个数字，即一个一位数，两位数就有两个 digit 了。所以隐含了 \square 、 \triangle 、 \star 都是小于 10 而大于 0 的整数，由(1) $\star=4$ ，且 $\square < \triangle$ ，则只有在 $\triangle=3$ 、 $\square=1$ 的情况下才可以满足上面的加法，所以(1)单独可回答问题，而(2) $\square=1$ ， \triangle 可取多个值，所以(2)单独不能回答问题，所以(A)is correct。

例 5: How many positive integers less than 20 are equal to the sum of a positive multiple of 3 and a positive multiple of 4?

- (A) Two
(B) Five
(C) Seven
(D) Ten
(E) Nineteen

翻译: 有多少小于 20 的正整数等于 3 的正倍数与 4 的正倍数的和?

解答: $x = 3m + 4n$ ($m, n \geq 1$)

表-7.2

m	n	x	m	n	x
1	1	7	2	2	14
1	2	11	2	3	18
1	3	15	3	1	13
1	4	19	3	2	17
2	1	10	4	1	16

所以共 10 个, (D) is correct。本题未必需数出 10 个, 当数出 7 个以上时, 则 10 就对了, 因为不可能是 19 个。

错误分析: 本题考查点即是列举法, 难度应该不是很大。但有人因为没有耐心没有答对, 也有的考生因为粗心答错题。

7.5.5 设“1”法

第 1 章中, 我们谈到了比例问题。比例问题最关键的是搞清分母是谁。有人喜欢将分母设为“ a ”或者“ x ”, 其实最简单的是将分母设为“1”, 即设“1”法。

设“1”法除了运算起来比较简便, 有时还会有意想不到的优点。

例 1: One-fifth of the light switches produced by a certain factory are defective. Four-fifths of the defective switches are rejected and $1/20$ of the nondefective switches are rejected by mistake. If all the switches not rejected are sold, what percent of the switches sold by the factory are defective?

- (A) 4%
(B) 5%
(C) 6.25%

(D) 11%

(E) 16%

翻译: 某工厂生产的 $1/5$ 的电灯开关是有缺陷的, $4/5$ 有缺陷的开关被剔除并且 $1/20$ 无缺陷的开关被错误地剔除。假如所有未剔除的开关都被出售, 该工厂出售的百分之多少开关是有缺陷的?

解答: 设总的开关为 1 (注意这不是开关的实际数目, 而是一个比值)。

出售的有缺陷的开关占总量的比率:

$$1/5 \times 1/5 = 1/25$$

出售的无缺陷开关占总量的比率:

$$4/5(1 - 1/20) = 19/25$$

则出售的开关中有缺陷的比例为:

$$\frac{1/25}{1/25 + 19/25} = 5\%$$

所以(B) is correct。

提示: 有人将总的开关数设为 x , 尽管 x 最终被约掉, 但这种设法无疑增加了计算量。

例 2: Forty percent of the rats included in an experiment were male rats. If some of the rats died during the experiment and 30 percent of the rats that died were male rats, what was the ratio of the death rate among the male rats to the death rate among the female rats?

- (A) $9/14$
(B) $3/4$
(C) $9/11$
(D) $6/7$
(E) $7/8$

翻译: 某一实验中包括的雄性老鼠占 40%。假如一些老鼠在实验中死亡, 并且 30% 死亡的老鼠是雄性老鼠, 雄性老鼠的死亡率与雌性老鼠的死亡率的比率为多少?

解答: 设共有 1 的老鼠死亡, 则雄性老鼠的

$$\text{死亡率为: } \frac{1 \times 30\%}{40\%} = \frac{3}{4}$$

雌性老鼠的死亡率为:

$$\frac{1 \times (1 - 30\%)}{1 - 40\%} = \frac{7}{6}$$

$$\text{所以死亡率的比率为: } \frac{\frac{3}{4}}{\frac{7}{6}} = \frac{9}{14}$$

所以(A) is correct.

错误分析: 有的读者会 Argue: 雌性老鼠的死亡率怎么会超过 100% 呢? 其实这与我们的对分母的设法有关, 我们设“共有 1 的老鼠死亡”, 实际等效于设“共有 $x\%$ 的老鼠死亡”。尽管所求的“雌性老鼠的死亡率 = $\frac{7}{6}$ ”并没有实际意义,

由于结果要求的是比率, 这并不影响结果。而设“1”法很大程度上简化了计算。

7.5.6 排除法

GMAT 中有很多选项设置得非常巧妙, 考生若能够抓住选项的特点, 运用排除法等技巧, 可以非常容易地求解。

例 1: At a certain state university last term, there were p students each of whom paid either the full tuition of x dollars or half the full tuition. What percent of the tuition paid by the p students last term was tuition from students who paid the full tuition?

- (1) Of the p students, 20 percent paid the full tuition.
- (2) The p students paid a total of \$91.2 million for tuition last term.

翻译: 在某一州立大学的上一学期, 有 p 个学生, 他们或者付全额学费 x 美元或者付半额学费, 在 p 个学生所付的学费中付全额学费的学生所付的学费所占的百分比是多少?

- (1) 在 p 个学生中, 20% 的人付全额学费。
- (2) p 个学生上学期共付了 91.2×10^6 美元的学费。

解答: 由(1)中付全额学费的学生有 20%, 设每人付 x 美元, 则付全额学费的学生所付的学费所占的百分比为:

$$\frac{20\%x}{20\%x + 80\% \times \frac{1}{2}x}$$

所以(1)可以单独回答上面的问题; (2)中已知所付学费的总量无法回答上面的问题。

所以(A) is correct.

错误分析: 题目中用的是字符, 条件(1)给的是比例, 而条件(2)给的是具体的数字, 条件(2)实际上根本不用考虑。有的考生花费了很长时间去考虑条件(2), 并且错误地结论条件(1)加上条件(2)可以求解, 选择了选项 C。

提示: 与题目不相关的条件, 如题目中用的是字符, 条件中给的是具体的数字, 不应该在考虑之列。

例 2: If x is an integer, is $(x + p)(x + q)$ an even integer?

- (1) q is an even integer.
- (2) p is an even integer.

翻译: 假如 x 是一个整数, $(x + p)(x + q)$ 是一个偶数吗?

- (1) q 是一个偶数。
- (2) p 是一个偶数。

解答: (1)中 q 是偶数, 但无法确定 $(x + p)$

和 $(x+q)$ 是否为偶数；(2)同理也无法确定；(1)+(2)同样无法确定 $(x+p)$ 、 $(x+q)$ 中是否至少有一个是偶数，

所以(E) is correct。

提示：细心的考生应该注意到题目中 p 和 q 是对称的，而条件(1)和条件(2)也相同。因此，选项(A)、(B)和(D)应该排除。因为若选项(A)成立，则选项(B)一定成立；反之亦然。若(A)和(B)都成立，则选项(D)也成立。排除了选项(A)、(B)和(D)，考生只需考虑选项(C)和(E)，即条件(1)和(2)联立是否能够得到结果。

例 3: If \otimes denotes a mathematical operation, does $x \otimes y = y \otimes x$ for all x and y ?

(1) For all x and y , $x \otimes y = 2(x^2 + y^2)$

(2) For all y , $0 \otimes y = 2y^2$

翻译：假如 \otimes 定义了一个算术运算，对于所有的 x 和 y ， $x \otimes y = y \otimes x$ 吗？

(1) 对于所有的 x 和 x 和 y ，

$$x \otimes y = 2(x^2 + y^2)$$

(2) 对于所有的 y ， $0 \otimes y = 2y^2$

解答：由(1)中的对 \otimes 运算的定义可以回答上面的问题为 Yes，所以(1)可以单独回答问题；对于(2)无法回答上面的问题，所以(A) is correct。

提示：条件(2)只定义了 y ，一定不可能单独成立。

7.5.7 十字交叉法

本小节将从中国古代经典的数学问题——鸡兔同笼问题出发，引出十字交叉法，然后举例说明十字交叉法在 GMAT 中的应用。

例 1: 鸡兔同笼问题

鸡兔共有 40 只，腿有 100 只。问鸡有多少只，兔有多少只。

解答：传统的算法是设鸡、兔分别有 x 和 y 只，然后列二元一次方程组求解。如图 7.10 所示。

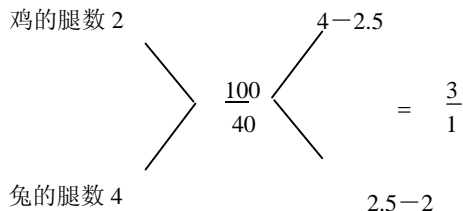


图 7.10

即鸡与兔的数目之比是 3:1。因此鸡有 $40 \times \frac{3}{3+1} = 30$ 只，兔有 $40 \times \frac{1}{3+1} = 10$ 只。

提示：十字交叉法正是用来处理二元混合物的。若有二元 A 和 B，A 的某一属性的值为 x ，B 的同一属性的值为 y ，若 A 和 B 混合物这一属性的平均值已知，则 A 与 B 的比可以通过十字交叉法求得。若 A 和 B 的总数已知，A 和 B 各自的数目也可以求得。

例 2: A haberdasher(男子服饰经销商)sells neckties for \$ 7 each and shirts for \$12 each. If he sells \$ 110 worth of ties and shirts and the total number of ties and shirts is 10, what is the amount of ties he sells?

(A) 2

(B) 3

(C) 4

(D) 5

(E) Can not be determined

翻译：一个男子服饰经销商出售领带和衬衫，领带价格为每件 7 美元，衬衫价格为每件 12 美元。如果他卖了 10 件东西，

总共卖了 110 美元,问他卖了多少条领带?

解答: 方法 1: 设卖了 x 条领带, y 件衬衫。有人认为即可得一个二元一次方程组, 因此可以求解。实际上两个方程分别为:

$$7x + 12y = 110$$

$$x + y = 10$$

可以求得 $x=2, y=8$

所以(A)is correct.

方法 2: 十字交叉法, 如图 7.11 所示。

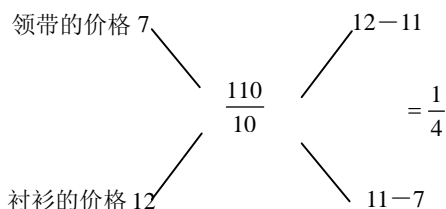


图 7.11

即领带和衬衫的数目之比是 1:4。因此领带有 $10 \times \frac{1}{1+4} = 2$ 条, 衬衫有

$$10 \times \frac{4}{1+4} = 8 \text{ 件。}$$

例 3: Committee X and Committee Y , which have no common members, will combine to form Committee Z . Does Committee X have more members than Committee Y ?

(1) The average (arithmetic mean) age of the members of Committee X is 25.7 years and the average age of the members of Committee Y is 29.3 years.

(2) The average (arithmetic mean) age of the members of Committee Z will be 26.6 years.

翻译: 委员会 X 和委员会 Y , 没有共同成员,

将组合成委员会 Z 。委员会 X 的成员比委员会 Y 的成员的数目多吗?

(1) X 委员会成员的平均年龄是 25.7 岁, Y 委员会成员的平均年龄是 29.3 岁。

(2) Z 委员会成员的平均年龄将为 26.6 岁。

解答: 方法 1: 由(1)中的两个委员会成员的平均年龄说明不了哪一个委员会的人数多; 由(2)中 Z 的平均年龄 26.6 岁也无法说明哪一个委员会成员多; (1)+(2)可以得到答案。因为若委员会 X 和 Y 人数相同, 则平均年龄应为 $(25.7+29.3)/2=27.5$ 岁, 而实际上委员会 Z 的平均年龄为 $26.6 < 27.5$, 则委员会 X 的人数一定多于委员会 Y 。所以 (C)is correct.

方法 2: 十字交叉法, 如图 7.12 所示: 单独由条件(1)和条件(2)都无法求解, (1)+(2)可以得到答案。

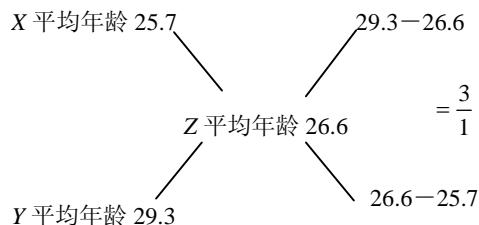


图 7.12

所以(C)is correct.

提示: 尽管 Data Sufficiency 题目并不需要精确求解, 掌握十字交叉法还是非常必要的。注意本题的一个必要条件是“委员会 X 和委员会 Y , 没有共同成员”。

7.5.8 特殊值法

例 1: If $x < y < z$ and $y - x > 5$, where x is an even integer and y and z are odd integers, what is the least possible value of $z - x$?

- (A) 6
(B) 7
(C) 8
(D) 9
(E) 10

翻译: 假如 $x < y < z$ 且 $y - x > 5$, x 是一个偶数并且 y 和 z 是奇数, $z - x$ 的最小值是多少?

解答: 若求 $z - x$ 的最小值, 则 y 也应取最小值, 而 $y - x > 5$, 且 x 为偶数, y, z 为奇数, 例如任取 $x = 2$, 则 y 最小值应为 9, 则 z 的最小值应为 11, 则 $z - x$ 的最小值应为 9。

所以(D) is correct。

错误分析: 由于本题 x, y, z 没有确定的值, 很多考生都无法下手。对于此类题目, 固定其中一个值, 再根据条件确定其他值, 是一种非常必要的解题方法。

例 2: If $w, x, y,$ and z are non - negative integers, each less than 3, and $w(3^3) - x(3^2) + y(3) + z = 34$, then $w + z =$

- (A) 0
(B) 1
(C) 2
(D) 3
(E) 4

翻译: 假如 w, x, y 和 z 为非负整数, 每个都小于 3, 且 $w(3^3) - x(3^2) + y(3) + z = 34$, 那么 $w + z$ 的值是多少?

解答: 因为 w, x, y, z 为小于 3 的非负整数, 则其取值范围为 0、1、2, 因为上面的和为 34, 那么 $w=1$, 因为若 $w=0$, 则 x, y, z 均取最大值 2 也不可能使和为 34, 而 $w = 2$, 则 $2 \times 3^3 = 54$, 同样违反上面等式, 由 $w = 1$ 得: $9x - 37 + z = 7$

则 $x = 0, y = 2, z = 1$ 上面等式成立。

那么 $w + z = 2$ 。

所以(C) is correct。

提示: 由于“ w, x, y, z 为小于 3 的非负整数”的条件限制, 本题尽管只有一个方程和 4 个未知数, 却得到了求解。对 w 的求解, 是解决问题的关键。 w 这个特殊值确定以后, 整个问题就迎刃而解了。

例 3: Which of the following symbols should be substituted for \square to make both of the statements below true for all integers n such that $-2 < n \leq 3$?

$$4 - n \square 6$$

$$4 - n \square 5$$

- (A) \leq
(B) $<$
(C) $=$
(D) $>$
(E) \geq

翻译: 下列哪一个符号应当代替 \square 使下面两个式子对所有的满足 $-2 < n \leq 3$ 的整数成立?

解答: 本题选择一个特殊值即可。这个特殊值是 -1 , -1 是满足 $-2 < n \leq 3$ 的最小整数。若 -1 能够使表达式成立, 当 $-2 < n \leq 3$, n 一定也能使表达式成立。

提示: 对于不等式问题, 可以选择一个或者两个边界值作为特殊值, 从而简化了问题的计算。

7.5.9 归纳法

用数学归纳法证明一个与自然数有关的命题的步骤是:

- (1) 证明当 n 取第一个值 n_0 时结论正确;
- (2) 假设当 $n=k(k \geq n_0)$ 时结论正确, 证

明当 $n=k+1$ 时结论也正确。

以上是用数学归纳法的步骤,在考试中虽然并不要求用数学归纳法去解题,但是却会考到对数学归纳法的概念的理解。

例 1: If P is a set of integers and 3 is in P , is every positive multiple of 3 in P ?

(1) If x is in P , then $x + 3$ is in P .

(2) If x is in P , then $x - 3$ is in P .

翻译: 假如 P 是一个整数集合并且包含 3,那么 3 的每个正倍数都在 P 中吗?

(1) 假如 x 在 P 中,则 $x + 3$ 也在 P 中

(2) 假如 x 在 P 中,则 $x - 3$ 也在 P 中

解答: 因 3 已在这一集合中,而 3 是 3 的 1 倍,实际上完成了上述数学归纳法中的第(1)步,这时 I 中指出当 n 在集合中, $n+3$ 也在集合中,由此完成了上述数学归纳法中的第(2)步,因从 3~6 在,从 6~9 在……,所有是 3 的倍数的自然数就都在集合中,而 II 中与数学归纳法中第(2)步指出的正好相反了。

所以(A)is correct.

例 2: What is the least number of digits (including repetitions) needed to express 10^{100} in decimal notation?

(A) 4

(B) 100

(C) 101

(D) 1,000

(E) 1,001

翻译: 以十进制计数来表达 10^{100} 需要用的最少的数字是多少(包括重复)?

解答: 本题的问题在于许多考生无法正确理解题意,decimal 有两个意思:十进制,小数。本题中的 decimal notation 指十进制计数,即通常使用的计数方式如 100,1000,10000。而另一种计数方式为:

scientific notation 科学计数法,指数字被表达为 10 的次方关系,如 1.0×10^2 , 1.7×10^3 等。本题问十进制计数表达 10^{100} 所需要的最少数目的数字,利用数学归纳法, $10^1=10$ 需要 2 个数字, $10^2=100$ 需要 3 个数字, $10^3=1000$ 需要 4 个数字,而 10^{100} 需要 101 个数字,所以(C)is correct.

提示: 与本题相似,很多无法下手的问题都可以通过数学归纳法求解。

7.6 本章习题精选

1. Is quadrilateral $RSTV$ a rectangle?

(1) The measure of $\angle RST$ is 90°

(2) The measure of $\angle TVR$ is 90°

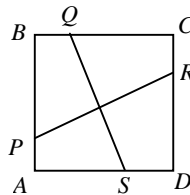


图 7.13

2. The figure above shows four pieces of tile that have been glued together to form a square tile $ABCD$. Is $PR=QS$?

(1) $BQ = CR = DS = AP$

(2) The perimeter of $ABCD$ is 16.

3. If a bottle is to be selected at random from a certain collection of bottles, what is the probability that the bottle will be defective?

(1) The ratio of the number of bottles in the collection that are defective to the number that are not defective is 3:500.

(2) The collection contains 3,521 bottles.

4. At a business association conference, the registration fee for members of the association was \$20 and the registration fee for nonmembers was \$25. If the total receipts from registration were \$5,500, did more members than nonmembers pay the registration fee?
- (1) Registration receipts from members were \$500 greater than receipts from nonmembers.
- (2) A total of 250 people paid the registration fee.
5. If x and y are integers between 10 and 99, inclusive, is $\frac{x-y}{9}$ an integer?
- (1) x and y have the same two digits, but in reverse order.
- (2) The tens' digit of x is 2 more than the units digit, and the tens digit of y is 2 less than the units digit.
7. If x and y are integers and $xy=5$, then $(x+y)^2=?$
- (A) 13
(B) 16
(C) 25
(D) 26
(E) 36
8. The average (arithmetic mean) of 3 different positive integers is 100 and the largest of these 3 integers is 120, what is the least possible value of the smallest of these 3 integers?
- (A) 1
(B) 10
(C) 61
(D) 71
(E) 80
9. How many two-digit whole numbers yield a remainder of 1 when divided by 10 and also yield a remainder of 1 when divided by 6?
- (A) None
(B) One
(C) Two
(D) Three
(E) Four
10. A company bought a total of 60 computers and 20 printers to modernize billing operations. If the price of each computer was three times the price of each printer, what percent of the total cost of the purchase was the total cost of the printers?
- (A) 10%
(B) 11%
(C) 15%
(D) 20%
(E) 25%
11. A shipment of banners contains banners of two different shapes, triangular and square, and two different colors, red and green. In a particular shipment 26% of the banners are square and 35% of the banners are red. If 60% of the red banners in the shipment are square, what is the ratio of red triangular banners to green triangular banners?
- (A) $\frac{7}{50}$
(B) $\frac{3}{13}$
(C) $\frac{7}{30}$
(D) $\frac{13}{37}$

- (E) $\frac{35}{26}$
12. The owner of a boutique decides to calculate the percentage of customers who purchase hats. If 40 percent of the store's customers decide to purchase items, and of those customers 15 percent purchase hats, what percent of the store's customers purchase hats?
- (A) 4%
(B) 6%
(C) 15%
(D) 24%
(E) 55%
13. A store raised the price of an item by exactly 10 percent. Which of the following could NOT be the resulting price of the item?
- (A) \$5.50
(B) \$7.60
(C) \$11.00
(D) \$12.10
(E) \$75.90
14. If when a certain integer x is divided by 5 the remainder is 2, then each of the following could also be an integer EXCEPT
- (A) $\frac{x}{17}$
(B) $\frac{x}{11}$
(C) $\frac{x}{10}$
(D) $\frac{x}{6}$
(E) $\frac{x}{3}$
15. If C is the temperature in degrees Celsius and F is the temperature in degrees Fahrenheit, then the relationship between temperatures on the two scales is expressed by the equation $9C=5(F-32)$. On a day when the temperature extremes recorded at a certain weather station differed by 45 degrees on the Fahrenheit scale, by how many degrees did the temperature extremes differ on the Celsius scale?
- (A) $\frac{65}{9}$
(B) 13
(C) 25
(D) 45
(E) 81
16. A certain sporting goods retailer gets a shipment of x bicycles each month. It is estimated that the retailer keeps half a shipment of bicycles in storage at all times. If the cost of storing a bicycle for one month is \$0.25, approximately what is the retailer's annual cost, in dollars, for storing bicycles?
- (A) $6x$
(B) $\frac{3x}{2}$
(C) x
(D) $\frac{x}{4}$
(E) $\frac{x}{24}$

17. How many gallons of water must be mixed with 1 gallon of a 15-percent salt solution to obtain a 10-percent salt solution?
- (A) 0.50
(B) 0.67
(C) 1.00
(D) 1.50
(E) 2.00
18. The table above shows the distance, in kilometers, by the most direct route, between any two of the four cities, R , S , T , and U . For example, the distance between City R and City U is 62 kilometers. What is the value of x ?
- (1) By the most direct route, the distance between S and T is twice the distance between S and R .
- (2) by the most direct route, the distance between T and U is 1.5 times the distance between R and T .

	R	S	T	U
R	0	y	x	62
S	y	0	56	75
T	x	56	0	69
U	62	75	69	0

图 7.15

参考答案

1~5	EAADA
6~10	无ECDA
11~15	CBBCC
16~18	BAB

第三部分 练习题和模拟题

第8章 Problem Solving 练习题

- A project scheduled to be carried out over a single fiscal year has a budget of \$12,600, divided into 12 equal monthly allocations. At the end of the fourth month of that fiscal year, the total amount actually spent on the project was \$4,580. By how much was the project over its budget?
 - \$ 380
 - \$ 540
 - \$1,050
 - \$1,380
 - \$1,430
- If the sum of 5, 8, 12, and 15 is equal to the sum of 3, 4, x , and $x + 3$, what is the value of x ?
 - 14
 - 15
 - 16
 - 17
 - 18
- For which of the following values of n is $\frac{100 + n}{n}$ NOT an integer?
 - 1
 - 2
 - 3
 - 4
 - 5
- Rectangular Floors X and Y have equal area. If Floor X is 12 feet by 18 feet and Floor Y is 9 feet wide, what is the length of Floor Y, in feet?
 - $13\frac{1}{2}$
 - 18
 - $18\frac{3}{4}$
 - 21
 - 24

Number of Employees	Salary
5	\$20,000
4	\$22,000
8	\$25,000
3	\$30,000

- The table above shows the number of employees at each of four salary levels at Company X. What is the average (arithmetic mean) salary for the 20 employees?
 - \$23,500
 - \$23,750
 - \$23,900
 - \$24,125
 - \$24,250

6. A case contains c cartons. Each carton contains b boxes, and each box contains 100 paper clips. How many paper clips are contained in 2 cases?

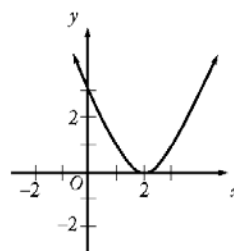
- (A) $100bc$
 (B) $\frac{100b}{c}$
 (C) $200bc$
 (D) $\frac{200b}{c}$
 (E) $\frac{200}{bc}$

7. The sum of prime numbers that are greater than 60 but less than 70 is

- (A) 67
 (B) 128
 (C) 191
 (D) 197
 (E) 260

8. A rainstorm increased the amount of water stored in State J reservoirs from 124 billion gallons to 138 billion gallons. If the storm increased the amount of water in the reservoirs to 82 percent of total capacity, approximately how many billion gallons of water were the reservoirs short of total capacity prior to the storm?

- (A) 9
 (B) 14
 (C) 25
 (D) 30
 (E) 44



9. On the graph above, when $x = \frac{1}{2}$, $y = 2$; and when $x = 1$, $y = 1$. The graph is symmetric with respect to the vertical line at $x = 2$. According to the graph, when $x = 3$, $y =$

- (A) -1
 (B) $-\frac{1}{2}$
 (C) 0
 (D) $\frac{1}{2}$
 (E) 1

10. When $\frac{1}{10}$ percent of 5,000 is subtracted from $\frac{1}{10}$ of 5,000, the difference is

- (A) 0
 (B) 50
 (C) 450
 (D) 495
 (E) 500

11. Which of the following is the value of $\sqrt[3]{0.000064}$?

- (A) 0.004
 (B) 0.008
 (C) 0.02
 (D) 0.04
 (E) 0.2

12. Raffle tickets numbered consecutively from 101 through 350 are placed in a box. What is the probability that a ticket selected at random will have a number with a hundreds digit of 2?
- (A) $\frac{2}{5}$
(B) $\frac{2}{7}$
(C) $\frac{33}{83}$
(D) $\frac{99}{250}$
(E) $\frac{100}{249}$
13. When Leo imported a certain item, he paid a 7 percent import tax on the portion of the total value of the item in excess of \$1,000. If the amount of the import tax that Leo paid was \$87.50, what was the total value of the item?
- (A) \$1,600
(B) \$1,850
(C) \$2,250
(D) \$2,400
(E) \$2,750
14. On Monday, a person mailed 8 packages weighing an average (arithmetic mean) of $12\frac{3}{8}$ pounds, and on Tuesday, 4 packages weighing an average of $15\frac{1}{4}$ pounds. What was the average weight, in pounds, of all the packages the person mailed on both days?
- (A) $13\frac{1}{3}$
(B) $13\frac{13}{16}$
(C) $15\frac{1}{2}$
(D) $15\frac{15}{16}$
(E) $16\frac{1}{2}$
15. $0.1 + (0.1)^2 + (0.1)^3 =$
- (A) 0.1
(B) 0.111
(C) 0.1211
(D) 0.2341
(E) 0.3
16. A carpenter constructed a rectangular sandbox with a capacity of 10 cubic feet. If the carpenter were to make a similar sandbox twice as long, twice as wide, and twice as high as the first sandbox, what would be the capacity, in cubic feet, of the second sandbox?
- (A) 20
(B) 40
(C) 60
(D) 80
(E) 100
17. A bakery opened yesterday with its daily supply of 40 dozen rolls. Half of the rolls were sold by noon, and 80 percent of the remaining rolls were sold between noon and closing time. How many dozen rolls had not been sold when the bakery closed yesterday?
- (A) 1
(B) 2
(C) 3
(D) 4
(E) 5
18. If the area of a square region having sides of length 6 centimeters is equal to the area of a rectangular region having width 2.5 centimeters, then the length of the rectangle, in centimeters, is
- (A) 8.5
(B) 9.5
(C) 9.6
(D) 10.5
(E) 14.4

19. 150 is what percent of 30?

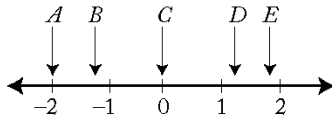
- (A) 5%
- (B) 20%
- (C) 50%
- (D) 200%
- (E) 500%

20. The ratio 2 to $\frac{1}{3}$ is equal to the ratio

- (A) 6 to 1
- (B) 5 to 1
- (C) 3 to 2
- (D) 2 to 3
- (E) 1 to 6

21. Running at the same constant rate, 6 identical machines can produce a total of 270 bottles per minute. At this rate, how many bottles could 10 such machines produce in 4 minutes?

- (A) 648
- (B) 1,800
- (C) 2,700
- (D) 10,800
- (E) 64,800



22. Of the five coordinates associated with points A, B, C, D, and E on the number line above, which has the greatest absolute value?

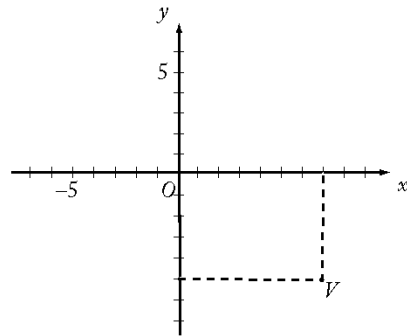
- (A) A
- (B) B
- (C) C
- (D) D
- (E) E

23. If n is a prime number greater than 3, what is the remainder when n^2 is divided by 12?

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) 5

24. $\frac{1}{1+\frac{1}{3}} - \frac{1}{1+\frac{1}{2}} =$

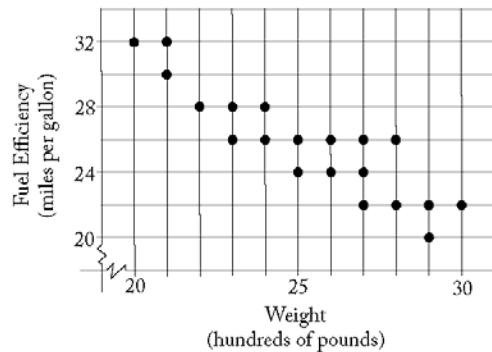
- (A) $-\frac{1}{3}$
- (B) $-\frac{1}{6}$
- (C) $-\frac{1}{12}$
- (D) $\frac{1}{12}$
- (E) $\frac{1}{3}$



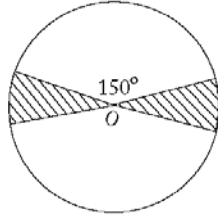
25. In the figure above, the coordinates of point V are

- (A) (-7,5)
- (B) (-5,7)
- (C) (5,7)
- (D) (7,5)
- (E) (7,-5)

26. A rope 40 feet long is cut into two pieces. If one piece is 18 feet longer than the other, what is the length, in feet, of the shorter piece?
- (A) 9
(B) 11
(C) 18
(D) 22
(E) 29
27. A student's average (arithmetic mean) test score on 4 tests is 78. What must be the student's score on a 5th test for the student's average score on the 5 tests to be 80?
- (A) 80
(B) 82
(C) 84
(D) 86
(E) 88
28. The average distance between the Sun and a certain planet is approximately 2.3×10^{14} inches. Which of the following is closest to the average distance between the Sun and the planet, in kilometers? (1 kilometer is approximately 3.9×10^4 inches.)
- (A) 7.1×10^8
(B) 5.9×10^9
(C) 1.6×10^{10}
(D) 1.6×10^{11}
(E) 5.9×10^{11}
29. If the quotient $\frac{a}{b}$ is positive, which of the following must be true?
- (A) $a > 0$
(B) $b > 0$
(C) $ab > 0$
(D) $a - b > 0$
(E) $a + b > 0$

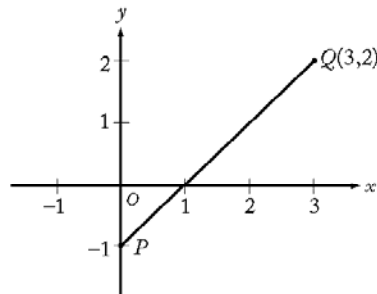


30. The dots on the graph above indicate the weights and fuel efficiency ratings for 20 cars. How many of the cars weigh more than 2,500 pounds and also get more than 22 miles per gallon?
- (A) 3
(B) 5
(C) 8
(D) 10
(E) 11
31. How many minutes does it take John to type y words if he types at the rate of x words per minute?
- (A) $\frac{x}{y}$
(B) $\frac{y}{x}$
(C) xy
(D) $\frac{60x}{y}$
(E) $\frac{y}{60x}$
32. $\sqrt{(16)(20) + (8)(32)} =$
- (A) $4\sqrt{20}$
(B) 24
(C) 25
(D) $4\sqrt{20} + 8\sqrt{2}$
(E) 32



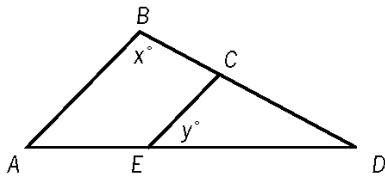
33. If O is the center of the circle above, what fraction of the circular region is shaded?
- (A) $\frac{1}{12}$
 (B) $\frac{1}{9}$
 (C) $\frac{1}{6}$
 (D) $\frac{1}{4}$
 (E) $\frac{1}{3}$
34. If Juan takes 11 seconds to run y yards, how many seconds will it take him to run x yards at the same rate?
- (A) $\frac{11x}{y}$
 (B) $\frac{11y}{x}$
 (C) $\frac{x}{11y}$
 (D) $\frac{11}{xy}$
 (E) $\frac{xy}{11}$
35. John has 10 pairs of matched socks. If he loses 7 individual socks, what is the greatest number of pairs of matched socks he can have left?
- (A) 7
 (B) 6
 (C) 5
 (D) 4
 (E) 3

36. What is the lowest positive integer that is divisible by each of the integers 1 through 7, inclusive?
- (A) 420
 (B) 840
 (C) 1,260
 (D) 2,520
 (E) 5,040
37. $\frac{1}{0.75-1} =$
- (A) -4
 (B) -0.25
 (C) 0.25
 (D) 0.75
 (E) 4
38. If $\frac{1.5}{0.2+x} = 5$, then $x =$
- (A) -3.7
 (B) 0.1
 (C) 0.3
 (D) 0.5
 (E) 2.8



39. In the figure above, the point on segment PQ that is twice as far from P as from Q is
- (A) (3,1)
 (B) (2,1)
 (C) (2,-1)
 (D) (1.5,0.5)
 (E) (1,0)

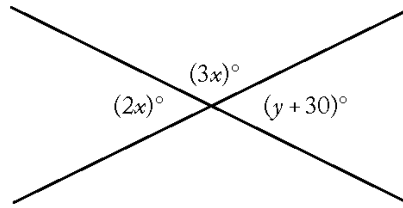
40. If n is an integer, which of the following must be even?
- (A) $n + 1$
(B) $n + 2$
(C) $2n$
(D) $2n + 1$
(E) n^2
41. If 4 is one solution of the equation $x^2 + 3x + k = 10$, where k is a constant, what is the other solution?
- (A) -7
(B) -4
(C) -3
(D) 1
(E) 6
42. If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ for all numbers a , b , c , and d , then
- $$\begin{vmatrix} 3 & 5 \\ -2 & 4 \end{vmatrix} =$$
- (A) -22
(B) -2
(C) 2
(D) 7
(E) 22
43. The sum $\frac{7}{8} + \frac{1}{9}$ is between
- (A) $\frac{1}{2}$ and $\frac{3}{4}$
(B) $\frac{3}{4}$ and 1
(C) 1 and $1\frac{1}{4}$
(D) $1\frac{1}{4}$ and $1\frac{1}{2}$
(E) $1\frac{1}{2}$ and 2
44. If $x = 1 - 3t$ and $y = 2t - 1$, then for what value of t does $x = y$?
- (A) $\frac{5}{2}$
(B) $\frac{3}{2}$
(C) $\frac{2}{3}$
(D) $\frac{2}{5}$
(E) 0
45. $1 - \left(\frac{1}{2} - \frac{2}{3}\right) =$
- (A) $\frac{6}{5}$
(B) $\frac{7}{6}$
(C) $\frac{6}{7}$
(D) $\frac{5}{6}$
(E) 0
46. $\frac{(0.3)^5}{(0.3)^3} =$
- (A) 0.001
(B) 0.01
(C) 0.09
(D) 0.9
(E) 1.0
47. In a horticultural experiment, 200 seeds were planted in plot I and 300 were planted in plot II. If 57 percent of the seeds in plot I germinated and 42 percent of the seeds in plot II germinated, what percent of the total number of planted seeds germinated?
- (A) 45.5%
(B) 46.5%
(C) 48.0%
(D) 49.5%
(E) 51.0%



Note: Figure not drawn to scale.

48. In the figure above, if $\overline{AB} \parallel \overline{CE}$, $CE = DE$, and $y = 45$, then $x =$
- (A) 45
 (B) 60
 (C) 67.5
 (D) 112.5
 (E) 135
49. How many integers n are there such that $1 < 5n + 5 < 25$?
- (A) Five
 (B) Four
 (C) Three
 (D) Two
 (E) One
50. If y is an integer, then the least possible value of $|23 - 5y|$ is
- (A) 1
 (B) 2
 (C) 3
 (D) 4
 (E) 5
51. $(\sqrt{7} + \sqrt{7})^2 =$
- (A) 98
 (B) 49
 (C) 28
 (D) 21
 (E) 14

52. In a certain population, there are 3 times as many people aged 21 or under as there are people over 21. The ratio of those 21 or under to the total population is
- (A) 1 to 2
 (B) 1 to 3
 (C) 1 to 4
 (D) 2 to 3
 (E) 3 to 4



53. In the figure above, the value of y is
- (A) 6
 (B) 12
 (C) 24
 (D) 36
 (E) 42
54. $\sqrt{80} + \sqrt{125} =$
- (A) $9\sqrt{5}$
 (B) $20\sqrt{5}$
 (C) $41\sqrt{5}$
 (D) $\sqrt{205}$
 (E) 100
55. Kelly and Chris packed several boxes with books. If Chris packed 60 percent of the total number of boxes, what was the ratio of the number of boxes Kelly packed to the number of boxes Chris packed?
- (A) 1 to 6
 (B) 1 to 4
 (C) 2 to 5
 (D) 3 to 5
 (E) 2 to 3

56. Of the following, which is the closest approximation of $\frac{50.2 \times 0.49}{199.8}$?

(A) $\frac{1}{10}$
 (B) $\frac{1}{8}$
 (C) $\frac{1}{4}$
 (D) $\frac{5}{4}$
 (E) $\frac{25}{2}$

57. The average (arithmetic mean) of 10, 30, and 50 is 5 more than the average of 20, 40, and

(A) 15
 (B) 25
 (C) 35
 (D) 45
 (E) 55

$$y = kx + 3$$

58. In the equation above, k is a constant. If $y = 17$ when $x = 2$, what is the value of y when $x = 4$?

(A) 34
 (B) 31
 (C) 14
 (D) 11
 (E) 7

59. Each week, Harry is paid x dollars per hour for the first 30 hours and $1.5x$ dollars for each additional hour worked that week. Each week, James is paid x dollars per hour for the first 40 hours and $2x$ dollars for each additional hour worked that week. Last week James worked a total of 41 hours. If Harry and James were paid the same amount last week, how many hours did Harry work last week?

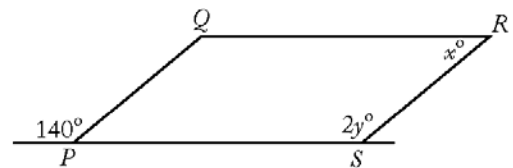
(A) 35
 (B) 36
 (C) 37
 (D) 38
 (E) 39

60. A glass was filled with 10 ounces of water, and 0.01 ounce of the water evaporated each day during a 20-day period. What percent of the original amount of water evaporated during this period?

(A) 0.002%
 (B) 0.02%
 (C) 0.2%
 (D) 2%
 (E) 20%

61. A glucose solution contains 15 grams of glucose per 100 cubic centimeters of solution. If 45 cubic centimeters of the solution were poured into an empty container, how many grams of glucose would be in the container?

(A) 3.00
 (B) 5.00
 (C) 5.50
 (D) 6.50
 (E) 6.75



62. In the figure above, if $PQRS$ is a parallelogram, then $y - x =$

(A) 30
 (B) 35
 (C) 40
 (D) 70
 (E) 100

63. If 1 kilometer is approximately 0.6 mile, which of the following best approximates the number of kilometers in 2 miles?

(A) $\frac{10}{3}$
(B) 3
(C) $\frac{6}{5}$
(D) $\frac{1}{3}$
(E) $\frac{3}{10}$

64. Lucy invested \$10,000 in a new mutual fund account exactly three years ago. The value of the account increased by 10 percent during the first year, increased by 5 percent during the second year, and decreased by 10 percent during the third year. What is the value of the account today?

(A) \$10,350
(B) \$10,395
(C) \$10,500
(D) \$11,500
(E) \$12,705

65. A certain fruit stand sold apples for \$0.70 each and bananas for \$0.50 each. If a customer purchased both apples and bananas from the stand for a total of \$6.30, what total number of apples and bananas did the customer purchase?

(A) 10
(B) 11
(C) 12
(D) 13
(E) 14

66. At a certain school, the ratio of the number of second graders to the number of fourth graders is 8 to 5, and the ratio of the number of first graders to the number of second graders is 3 to 4. If the ratio of the number of third graders to the number of fourth graders is 3 to 2, what is the ratio of the number of first graders to the number of third graders?

(A) 16 to 15
(B) 9 to 5
(C) 5 to 16
(D) 5 to 4
(E) 4 to 5

$$A = \{2, 3, 4, 5\}$$

$$B = \{4, 5, 6, 7, 8\}$$

67. Two integers will be randomly selected from the sets above, one integer from set A and one integer from set B. What is the probability that the sum of the two integers will equal 9?

(A) 0.15
(B) 0.20
(C) 0.25
(D) 0.30
(E) 0.33

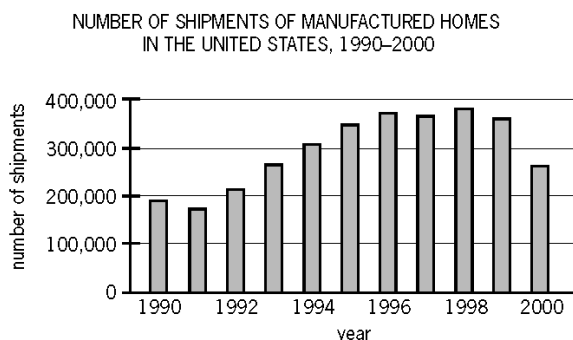
68. At a certain instant in time, the number of cars, N , traveling on a portion of a certain highway can be estimated by the formula

$$N = \frac{20Ld}{600 + s^2}$$

where L is the number of lanes in the same direction, d is the length of the portion of the highway, in feet, and s is the average speed of the cars, in miles per hour. Based on the formula, what is the estimated number of cars traveling on a $\frac{1}{2}$ -mile portion of the

highway if the highway has 2 lanes in the same direction and the average speed of the cars is 40 miles per hour? (5,280 feet = 1 mile)

(A) 155
(B) 96
(C) 80
(D) 48
(E) 24



69. According to the chart shown, which of the following is closest to the median annual number of shipments of manufactured homes in the United States for the years from 1990 to 2000, inclusive?
- (A) 250,000
(B) 280,000
(C) 310,000
(D) 325,000
(E) 340,000
70. If $y\left(\frac{3x-5}{2}\right) = y$ and $y \neq 0$, then $x =$
- (A) $\frac{2}{3}$
(B) $\frac{5}{3}$
(C) $\frac{7}{3}$
(D) 1
(E) 4
71. If $x + 5 > 2$ and $x - 3 < 7$, the value of x must be between which of the following pairs of numbers?
- (A) -3 and 10
(B) -3 and 4
(C) 2 and 7
(D) 3 and 4
(E) 3 and 10
72. A gym class can be divided into 8 teams with an equal number of players on each team or into 12 teams with an equal number of players on each team. What is the lowest possible number of students in the class?
- (A) 20
(B) 24
(C) 36
(D) 48
(E) 96
73. If $r = 0.345$, $s = (0.345)^2$, and $t = \sqrt{0.345}$, which of the following is the correct ordering of r , s , and t ?
- (A) $r < s < t$
(B) $r < t < s$
(C) $s < t < r$
(D) $s < r < t$
(E) $t < r < s$
74. A total of n trucks and cars are parked in a lot. If the number of cars is $\frac{1}{4}$ the number of trucks, and $\frac{2}{3}$ of the trucks are pickups, how many pickups, in terms of n , are parked in the lot?
- (A) $\frac{1}{6}n$
(B) $\frac{5}{12}n$
(C) $\frac{1}{2}n$
(D) $\frac{8}{15}n$
(E) $\frac{11}{12}n$
75. At least $\frac{2}{3}$ of the 40 members of a committee must vote in favor of a resolution for it to pass. What is the greatest number of members who could vote against the resolution and still have it pass?
- (A) 19
(B) 17
(C) 16
(D) 14
(E) 13

76. In the Johnsons' monthly budget, the dollar amounts allocated to household expenses, food, and miscellaneous items are in the ratio 5:2:1, respectively. If the total amount allocated to these three categories is \$1,800, what is the amount allocated to food?

(A) \$900
 (B) \$720
 (C) \$675
 (D) \$450
 (E) \$225

77. There are 4 more women than men on Centerville's board of education. If there are 10 members on the board, how many are women?

(A) 3
 (B) 4
 (C) 6
 (D) 7
 (E) 8

78. Leona bought a 1-year, \$10,000 certificate of deposit that paid interest at an annual rate of 8 percent compounded semiannually. What was the total amount of interest paid on this certificate at maturity?

(A) \$10,464
 (B) \$ 864
 (C) \$ 816
 (D) \$ 800
 (E) \$ 480

79.
$$\frac{(0.0036)(2.8)}{(0.04)(0.1)(0.003)} =$$

(A) 840.0
 (B) 84.0
 (C) 8.4
 (D) 0.84
 (E) 0.084

80. Machine A produces bolts at a uniform rate of 120 every 40 seconds, and Machine B produces bolts at a uniform rate of 100 every 20 seconds. If the two machines run simultaneously, how many seconds will it take for them to produce a total of 200 bolts?

(A) 22
 (B) 25
 (C) 28
 (D) 32
 (E) 56

Time	Amount
1:00 P.M.	10.0 grams
4:00 P.M.	x grams
7:00 P.M.	14.4 grams

81. Data for a certain biology experiment are given in the table above. If the amount of bacteria present increased by the same factor during each of the two 3-hour periods shown, how many grams of bacteria were present at 4:00 P.M. ?

(A) 12.0
 (B) 12.1
 (C) 12.2
 (D) 12.3
 (E) 12.4

82. If n is an integer greater than 6, which of the following must be divisible by 3 ?

(A) $n(n + 1)(n - 4)$
 (B) $n(n + 2)(n - 1)$
 (C) $n(n + 3)(n - 5)$
 (D) $n(n + 4)(n - 2)$
 (E) $n(n + 5)(n - 6)$

83. The total cost for Company X to produce a batch of tools is \$10,000 plus \$3 per tool. Each tool sells for \$8. The gross profit earned from producing and selling these tools is the total income from sales minus the total production cost. If a batch of 20,000 tools is produced and sold, then Company X's gross profit per tool is

(A) \$3.00
 (B) \$3.75
 (C) \$4.50
 (D) \$5.00
 (E) \$5.50

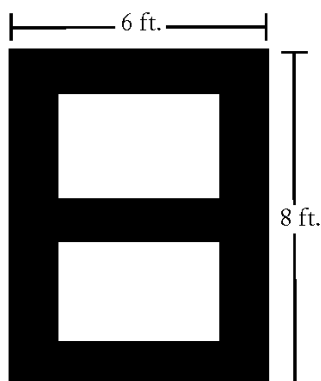
84. A dealer originally bought 100 identical batteries at a total cost of q dollars. If each battery was sold at 50 percent above the original cost per battery, then, in terms of q , for how many dollars was each battery sold?
- (A) $\frac{3q}{200}$
 (B) $\frac{3q}{2}$
 (C) $150q$
 (D) $\frac{q}{100} + 50$
 (E) $\frac{150}{q}$
85. In an increasing sequence of 10 consecutive integers, the sum of the first 5 integers is 560. What is the sum of the last 5 integers in the sequence?
- (A) 585
 (B) 580
 (C) 575
 (D) 570
 (E) 565
86. Machine A produces 100 parts twice as fast as Machine B does. Machine B produces 100 parts in 40 minutes. If each machine produces parts at a constant rate, how many parts does Machine A produce in 6 minutes?
- (A) 30
 (B) 25
 (C) 20
 (D) 15
 (E) 7.5
87. A necklace is made by stringing N individual beads together in the repeating pattern red bead, green bead, white bead, blue bead, and yellow bead. If the necklace design begins with a red bead and ends with a white bead, then N could equal
- (A) 16
 (B) 32
 (C) 41
 (D) 54
 (E) 68
88. In the xy -coordinate system, if (a, b) and $(a + 3, b + k)$ are two points on the line defined by the equation $x = 3y - 7$, then $k =$
- (A) 9
 (B) 3
 (C) $\frac{7}{3}$
 (D) 1
 (E) $\frac{1}{3}$
89. If s is the product of the integers from 100 to 200, inclusive, and t is the product of the integers from 100 to 201, inclusive, what is $\frac{1}{s} + \frac{1}{t}$ in terms of t ?
- (A) $\frac{(201)^2}{t}$
 (B) $\frac{(202)(201)}{t}$
 (C) $\frac{201}{t}$
 (D) $\frac{202}{t}$
 (E) $\frac{(202)(201)}{t^2}$
90. If Jake loses 8 pounds, he will weigh twice as much as his sister. Together they now weigh 278 pounds. What is Jake's present weight, in pounds?
- (A) 131
 (B) 135
 (C) 139
 (D) 147
 (E) 188

91. A certain store sells all maps at one price and all books at another price. On Monday the store sold 12 maps and 10 books for a total of \$38.00, and on Tuesday the store sold 20 maps and 15 books for a total of \$60.00. At this store, how much less does a map sell for than a book?
- (A) \$0.25
(B) \$0.50
(C) \$0.75
(D) \$1.00
(E) \$1.25
92. A store reported total sales of \$385 million for February of this year. If the total sales for the same month last year was \$320 million, approximately what was the percent increase in sales?
- (A) 2%
(B) 17%
(C) 20%
(D) 65%
(E) 83%
- List I: 3, 6, 8, 19
List II: x , 3, 6, 8, 19
93. If the median of the numbers in list I above is equal to the median of the numbers in list II above, what is the value of x ?
- (A) 6
(B) 7
(C) 8
(D) 9
(E) 10
94. In a certain city, 60 percent of the registered voters are Democrats and the rest are Republicans. In a mayoral race, if 75 percent of the registered voters who are Democrats and 20 percent of the registered voters who are Republicans are expected to vote for Candidate A, what percent of the registered voters are expected to vote for Candidate A?
- (A) 50%
(B) 53%
(C) 54%
(D) 55%
(E) 57%

95. $\frac{1}{2} + \left[\left(\frac{2}{3} \times \frac{3}{8} \right) \div 4 \right] - \frac{9}{16} =$
- (A) $\frac{29}{16}$
(B) $\frac{19}{16}$
(C) $\frac{15}{16}$
(D) $\frac{9}{13}$
(E) 0
96. Water consists of hydrogen and oxygen, and the approximate ratio, by mass, of hydrogen to oxygen is 2:16. Approximately how many grams of oxygen are there in 144 grams of water?
- (A) 16
(B) 72
(C) 112
(D) 128
(E) 142
97. If $x(2x + 1) = 0$ and $\left(x + \frac{1}{2}\right)(2x - 3) = 0$, then $x =$
- (A) -3
(B) $-\frac{1}{2}$
(C) 0
(D) $\frac{1}{2}$
(E) $\frac{3}{2}$

98. On a scale that measures the intensity of a certain phenomenon, a reading of $n + 1$ corresponds to an intensity that is 10 times the intensity corresponding to a reading of n . On that scale, the intensity corresponding to a reading of 8 is how many times as great as the intensity corresponding to a reading of 3?
- (A) 5
(B) 50
(C) 10^5
(D) 5^{10}
(E) $8^{10} - 3^{10}$
99. For the positive numbers, n , $n + 1$, $n + 2$, $n + 4$, and $n + 8$, the mean is how much greater than the median?
- (A) 0
(B) 1
(C) $n + 1$
(D) $n + 2$
(E) $n + 3$
100. If $T = \frac{5}{9}(K - 32)$, and if $T = 290$, then $K =$
- (A) $\frac{1,738}{9}$
(B) 322
(C) 490
(D) 554
(E) $\frac{2,898}{5}$
101. The water from one outlet, flowing at a constant rate, can fill a swimming pool in 9 hours. The water from a second outlet, flowing at a constant rate, can fill the same pool in 5 hours. If both outlets are used at the same time, approximately what is the number of hours required to fill the pool?
- (A) 0.22
(B) 0.31
(C) 2.50
(D) 3.21
(E) 4.56
102. If a square mirror has a 20-inch diagonal, what is the approximate perimeter of the mirror, in inches?
- (A) 40
(B) 60
(C) 80
(D) 100
(E) 120
103. The present ratio of students to teachers at a certain school is 30 to 1. If the student enrollment were to increase by 50 students and the number of teachers were to increase by 5, the ratio of students to teachers would then be 25 to 1. What is the present number of teachers?
- (A) 5
(B) 8
(C) 10
(D) 12
(E) 15
104. What is the smallest integer n for which $25^n > 5^{12}$?
- (A) 6
(B) 7
(C) 8
(D) 9
(E) 10
105. Sixty percent of the members of a study group are women, and 45 percent of those women are lawyers. If one member of the study group is to be selected at random, what is the probability that the member selected is a woman lawyer?
- (A) 0.10
(B) 0.15
(C) 0.27
(D) 0.33
(E) 0.45

106. When positive integer x is divided by positive integer y , the remainder is 9. If $\frac{x}{y} = 96.12$, what is the value of y ?
- (A) 96
(B) 75
(C) 48
(D) 25
(E) 12
107. If x is the product of the positive integers from 1 to 8, inclusive, and if i , k , m , and p are positive integers such that $x = 2^i 3^k 5^m 7^p$, then $i + k + m + p =$
- (A) 4
(B) 7
(C) 8
(D) 11
(E) 12
108. If $t = \frac{1}{2^9 \times 5^3}$ is expressed as a terminating decimal, how many zeros will t have between the decimal point and the first nonzero digit to the right of the decimal point?
- (A) Three
(B) Four
(C) Five
(D) Six
(E) Nine
109. A pharmaceutical company received \$3 million in royalties on the first \$20 million in sales of the generic equivalent of one of its products and then \$9 million in royalties on the next \$108 million in sales. By approximately what percent did the ratio of royalties to sales decrease from the first \$20 million in sales to the next \$108 million in sales?
- (A) 8%
(B) 15%
(C) 45%
(D) 52%
(E) 56%
110. If p is the product of the integers from 1 to 30, inclusive, what is the greatest integer k for which 3^k is a factor of p ?
- (A) 10
(B) 12
(C) 14
(D) 16
(E) 18
111. If candy bars that regularly sell for \$0.40 each are on sale at two for \$0.75, what is the percent reduction in the price of two such candy bars purchased at the sale price?
- (A) $2\frac{1}{2}\%$
(B) $6\frac{1}{4}\%$
(C) $6\frac{2}{3}\%$
(D) 8%
(E) $12\frac{1}{2}\%$
112. If $s > 0$ and $\sqrt{\frac{r}{s}} = s$, what is r in terms of s ?
- (A) $\frac{1}{s}$
(B) \sqrt{s}
(C) $s\sqrt{s}$
(D) s^3
(E) $s^2 - s$



113. The front of a 6-foot-by-8-foot rectangular door has brass rectangular trim, as indicated by the shading in the figure above. If the trim is uniformly 1 foot wide, what fraction of the door's front surface is covered by the trim?

- (A) $\frac{13}{48}$
 (B) $\frac{5}{12}$
 (C) $\frac{1}{2}$
 (D) $\frac{7}{12}$
 (E) $\frac{5}{8}$

114. If $a = -0.3$, which of the following is true?

- (A) $a < a^2 < a^3$
 (B) $a < a^3 < a^2$
 (C) $a^2 < a < a^3$
 (D) $a^2 < a^3 < a$
 (E) $a^3 < a < a^2$

115. Mary's income is 60 percent more than Tim's income, and Tim's income is 40 percent less than Juan's income. What percent of Juan's income is Mary's income?

- (A) 124%
 (B) 120%
 (C) 96%
 (D) 80%
 (E) 64%

	City A	City B	City C	City D	City E
City A		•	•	•	•
City B			•	•	•
City C				•	•
City D					•
City E					

116. Each • in the mileage table above represents an entry indicating the distance between a pair of the five cities. If the table were extended to represent the distances between all pairs of 30 cities and each distance were to be represented by only one entry, how many entries would the table then have?

- (A) 60
 (B) 435
 (C) 450
 (D) 465
 (E) 900

117. If n is positive, which of the following is equal to

$$\frac{1}{\sqrt{n+1} - \sqrt{n}} ?$$

- (A) 1
 (B) $\sqrt{2n+1}$
 (C) $\frac{\sqrt{n+1}}{\sqrt{n}}$
 (D) $\sqrt{n+1} - \sqrt{n}$
 (E) $\sqrt{n+1} + \sqrt{n}$

118. The ratio of the length to the width of a rectangular advertising display is approximately 3.3 to 2. If the width of the display is 8 meters, what is the approximate length of the display, in meters?

- (A) 7
 (B) 11
 (C) 13
 (D) 16
 (E) 26

119. Which of the following is equivalent to the pair of inequalities $x + 6 > 10$ and $x - 3 \leq 5$?
- (A) $2 \leq x < 16$
 (B) $2 \leq x < 4$
 (C) $2 < x \leq 8$
 (D) $4 < x \leq 8$
 (E) $4 \leq x < 16$
120. David has d books, which is 3 times as many as Jeff and $\frac{1}{2}$ as many as Paula. How many books do the three of them have altogether, in terms of d ?
- (A) $\frac{5}{6}d$
 (B) $\frac{7}{3}d$
 (C) $\frac{10}{3}d$
 (D) $\frac{7}{2}d$
 (E) $\frac{9}{2}d$
121. There are 8 teams in a certain league and each team plays each of the other teams exactly once. If each game is played by 2 teams, what is the total number of games played?
- (A) 15
 (B) 16
 (C) 28
 (D) 56
 (E) 64
122. An operation θ is defined by the equation $a \theta b = \frac{a - b}{a + b}$, for all numbers a and b such that $a \neq -b$. If $a \theta -c$ and $a \theta c = 0$, then $c =$
- (A) $-a$
 (B) $-\frac{1}{a}$
 (C) 0
 (D) $\frac{1}{a}$
 (E) a
123. The price of lunch for 15 people was \$207.00, including a 15 percent gratuity for service. What was the average price per person, EXCLUDING the gratuity?
- (A) \$11.73
 (B) \$12.00
 (C) \$13.80
 (D) \$14.00
 (E) \$15.87
124. In Town X, 64 percent of the population are employed, and 48 percent of the population are employed males. What percent of the employed people in Town X are females?
- (A) 16%
 (B) 25%
 (C) 32%
 (D) 40%
 (E) 52%
125. If $\frac{p}{q} < 1$, and p and q are positive integers, which of the following must be greater than 1?
- (A) $\sqrt{\frac{p}{q}}$
 (B) $\frac{p}{q^2}$
 (C) $\frac{p}{2q}$
 (D) $\frac{q}{p^2}$
 (E) $\frac{q}{p}$

126. It would take one machine 4 hours to complete a large production order and another machine 3 hours to complete the same order. How many hours would it take both machines, working simultaneously at their respective constant rates, to complete the order?

- (A) $\frac{7}{12}$
 (B) $1\frac{1}{2}$
 (C) $1\frac{5}{7}$
 (D) $3\frac{1}{2}$
 (E) 7

127. To mail a package, the rate is x cents for the first pound and y cents for each additional pound, where $x > y$. Two packages weighing 3 pounds and 5 pounds, respectively, can be mailed separately or combined as one package. Which method is cheaper, and how much money is saved?

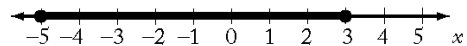
- (A) Combined, with a savings of $x - y$ cents
 (B) Combined, with a savings of $y - x$ cents
 (C) Combined, with a savings of x cents
 (D) Separately, with a savings of $x - y$ cents
 (E) Separately, with a savings of y cents

128. If money is invested at r percent interest, compounded annually, the amount of the investment will double in approximately $\frac{70}{r}$ years. If Pat's parents invested \$5,000 in a long-term bond that pays 8 percent interest, compounded annually, what will be the approximate total amount of the investment 18 years later, when Pat is ready for college?

- (A) \$20,000
 (B) \$15,000
 (C) \$12,000
 (D) \$10,000
 (E) \$ 9,000

129. On a recent trip, Cindy drove her car 290 miles, rounded to the nearest 10 miles, and used 12 gallons of gasoline, rounded to the nearest gallon. The actual number of miles per gallon that Cindy's car got on this trip must have been between

- (A) $\frac{290}{12.5}$ and $\frac{290}{11.5}$
 (B) $\frac{295}{12}$ and $\frac{285}{11.5}$
 (C) $\frac{285}{12}$ and $\frac{295}{12}$
 (D) $\frac{285}{12.5}$ and $\frac{295}{11.5}$
 (E) $\frac{295}{12.5}$ and $\frac{285}{11.5}$



130. Which of the following inequalities is an algebraic expression for the shaded part of the number line above?

- (A) $|x| \leq 3$
 (B) $|x| \leq 5$
 (C) $|x - 2| \leq 3$
 (D) $|x - 1| \leq 4$
 (E) $|x + 1| \leq 4$

131. A factory has 500 workers, 15 percent of whom are women. If 50 additional workers are to be hired and all of the present workers remain, how many of the additional workers must be women in order to raise the percent of women employees to 20 percent?

- (A) 3
 (B) 10
 (C) 25
 (D) 30
 (E) 35

132. In a small snack shop, the average (arithmetic mean) revenue was \$400 per day over a 10-day period. During this period, if the average daily revenue was \$360 for the first 6 days, what was the average daily revenue for the last 4 days?

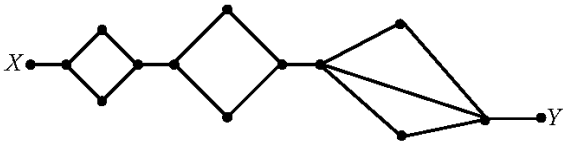
- (A) \$420
- (B) \$440
- (C) \$450
- (D) \$460
- (E) \$480

133. A certain country had a total annual expenditure of $\$1.2 \times 10^{12}$ last year. If the population of the country was 240 million last year, what was the per capita expenditure?

- (A) \$ 500
- (B) \$1,000
- (C) \$2,000
- (D) \$3,000
- (E) \$5,000

134. A certain rectangular window is twice as long as it is wide. If its perimeter is 10 feet, then its dimensions in feet are

- (A) $\frac{3}{2}$ by $\frac{7}{2}$
- (B) $\frac{5}{3}$ by $\frac{10}{3}$
- (C) 2 by 4
- (D) 3 by 6
- (E) $\frac{10}{3}$ by $\frac{20}{3}$



135. The diagram above shows the various paths along which a mouse can travel from point X, where it is released, to point Y, where it is rewarded with a food pellet. How many different paths from X to Y can the mouse take if it goes directly from X to Y without retracing any point along a path?

- (A) 6
- (B) 7
- (C) 12
- (D) 14
- (E) 17

136. If the operation \odot is defined by $x \odot y = \sqrt{xy}$ for all positive numbers x and y , then $(5 \odot 45) \odot 60 =$

- (A) 30
- (B) 60
- (C) 90
- (D) $30\sqrt{15}$
- (E) $60\sqrt{15}$

137. A bar over a sequence of digits in a decimal indicates that the sequence repeats indefinitely.

What is the value of $(10^4 - 10^2)(0.00\overline{12})$?

- (A) 0
- (B) $0.\overline{12}$
- (C) 1.2
- (D) 10
- (E) 12

138. At a loading dock, each worker on the night crew loaded $\frac{3}{4}$ as many boxes as each worker on the day crew. If the night crew has $\frac{4}{5}$ as many workers as the day crew, what fraction of all the boxes loaded by the two crews did the day crew load?

- (A) $\frac{1}{2}$
- (B) $\frac{2}{5}$
- (C) $\frac{3}{5}$
- (D) $\frac{4}{5}$
- (E) $\frac{5}{8}$

139. A restaurant meal cost \$35.50 and there was no tax. If the tip was more than 10 percent but less than 15 percent of the cost of the meal, then the total amount paid must have been between

(A) \$40 and \$42
 (B) \$39 and \$41
 (C) \$38 and \$40
 (D) \$37 and \$39
 (E) \$36 and \$37

140. In a weight-lifting competition, the total weight of Joe's two lifts was 750 pounds. If twice the weight of his first lift was 300 pounds more than the weight of his second lift, what was the weight, in pounds, of his first lift?

(A) 225
 (B) 275
 (C) 325
 (D) 350
 (E) 400

141. A club collected exactly \$599 from its members. If each member contributed at least \$12, what is the greatest number of members the club could have?

(A) 43
 (B) 44
 (C) 49
 (D) 50
 (E) 51

142. If y is the smallest positive integer such that 3,150 multiplied by y is the square of an integer, then y must be

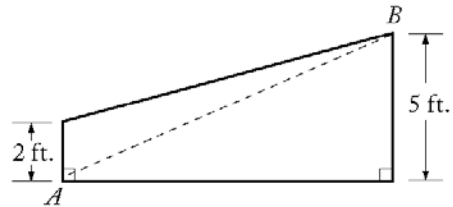
(A) 2
 (B) 5
 (C) 6
 (D) 7
 (E) 14

143. If $[x]$ is the greatest integer less than or equal to x , what is the value of $[-1.6] + [3.4] + [2.7]$?

(A) 3
 (B) 4
 (C) 5
 (D) 6
 (E) 7

144. If $\frac{4-x}{2+x} = x$, what is the value of $x^2 + 3x - 4$?

(A) -4
 (B) -1
 (C) 0
 (D) 1
 (E) 2

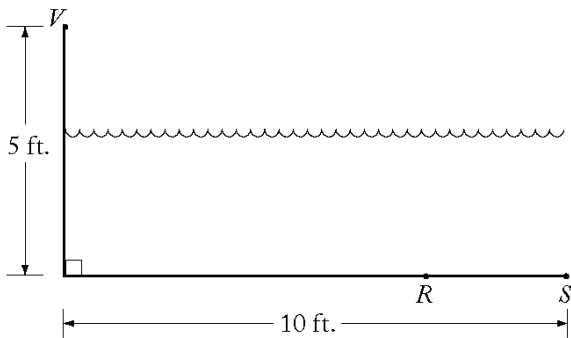


145. The trapezoid shown in the figure above represents a cross section of the rudder of a ship. If the distance from A to B is 13 feet, what is the area of the cross section of the rudder in square feet?

(A) 39
 (B) 40
 (C) 42
 (D) 45
 (E) 46.5

146. In a certain sequence, the term x_n is given by the formula $x_n = 2x_{n-1} - \frac{1}{2}(x_{n-2})$ for all $n \geq 2$. If $x_0 = 3$ and $x_1 = 2$, what is the value of x_3 ?

(A) 2.5
 (B) 3.125
 (C) 4
 (D) 5
 (E) 6.75



147. In the figure above, V represents an observation point at one end of a pool. From V , an object that is actually located on the bottom of the pool at point R appears to be at point S . If $VR = 10$ feet, what is the distance RS , in feet, between the actual position and the perceived position of the object?

- (A) $10 - 5\sqrt{3}$
 (B) $10 - 5\sqrt{2}$
 (C) 2
 (D) $2\frac{1}{2}$
 (E) 4

148. If x , y , and k are positive numbers such that

$$\left(\frac{x}{x+y}\right)(10) + \left(\frac{y}{x+y}\right)(20) = k \text{ and if } x < y,$$

which of the following could be the value of k ?

- (A) 10
 (B) 12
 (C) 15
 (D) 18
 (E) 30

149. During a trip, Francine traveled x percent of the total distance at an average speed of 40 miles per hour and the rest of the distance at an average speed of 60 miles per hour. In terms of x , what was Francine's average speed for the entire trip?

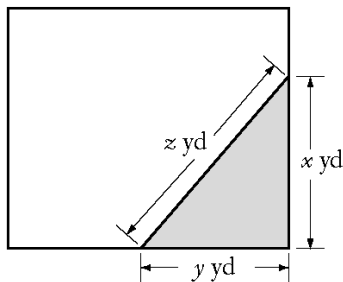
- (A) $\frac{180-x}{2}$
 (B) $\frac{x+60}{4}$
 (C) $\frac{300-x}{5}$
 (D) $\frac{600}{115-x}$
 (E) $\frac{12,000}{x+200}$

150. If $x = -1$, then $\frac{x^4 - x^3 + x^2}{x - 1} =$

- (A) $-\frac{3}{2}$
 (B) $-\frac{1}{2}$
 (C) 0
 (D) $\frac{1}{2}$
 (E) $\frac{3}{2}$

151. A toy store regularly sells all stock at a discount of 20 percent to 40 percent. If an additional 25 percent were deducted from the discount price during a special sale, what would be the lowest possible price of a toy costing \$16 before any discount?

- (A) \$ 5.60
 (B) \$ 7.20
 (C) \$ 8.80
 (D) \$ 9.60
 (E) \$15.20

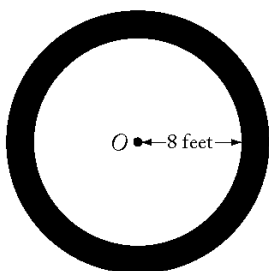


152. The shaded portion of the rectangular lot shown above represents a flower bed. If the area of the bed is 24 square yards and $x = y + 2$, then z equals
- (A) $\sqrt{13}$
 (B) $2\sqrt{13}$
 (C) 6
 (D) 8
 (E) 10
153. Jack is now 14 years older than Bill. If in 10 years Jack will be twice as old as Bill, how old will Jack be in 5 years?
- (A) 9
 (B) 19
 (C) 21
 (D) 23
 (E) 33
154. An empty pool being filled with water at a constant rate takes 8 hours to fill to $\frac{3}{5}$ of its capacity. How much more time will it take to finish filling the pool?
- (A) 5 hr 30 min
 (B) 5 hr 20 min
 (C) 4 hr 48 min
 (D) 3 hr 12 min
 (E) 2 hr 40 min
155. A positive number x is multiplied by 2, and this product is then divided by 3. If the positive square root of the result of these two operations equals x , what is the value of x ?
- (A) $\frac{9}{4}$
 (B) $\frac{3}{2}$
 (C) $\frac{4}{3}$
 (D) $\frac{2}{3}$
 (E) $\frac{1}{2}$
156. A tank contains 10,000 gallons of a solution that is 5 percent sodium chloride by volume. If 2,500 gallons of water evaporate from the tank, the remaining solution will be approximately what percent sodium chloride?
- (A) 1.25%
 (B) 3.75%
 (C) 6.25%
 (D) 6.67%
 (E) 11.7%
157. For any positive integer n , the sum of the first n positive integers equals $\frac{n(n+1)}{2}$. What is the sum of all the even integers between 99 and 301?
- (A) 10,100
 (B) 20,200
 (C) 22,650
 (D) 40,200
 (E) 45,150
158. A committee is composed of w women and m men. If 3 women and 2 men are added to the committee, and if one person is selected at random from the enlarged committee, then the probability that a woman is selected can be represented by

- (A) $\frac{w}{m}$
 (B) $\frac{w}{w+m}$
 (C) $\frac{w+3}{m+2}$
 (D) $\frac{w+3}{w+m+3}$
 (E) $\frac{w+3}{w+m+5}$

159. How many prime numbers between 1 and 100 are factors of 7,150?

- (A) One
 (B) Two
 (C) Three
 (D) Four
 (E) Five



160. The figure above shows a circular flower bed, with its center at O , surrounded by a circular path that is 3 feet wide. What is the area of the path, in square feet?

- (A) 25π
 (B) 38π
 (C) 55π
 (D) 57π
 (E) 64π

161. The positive integer n is divisible by 25. If \sqrt{n} is greater than 25, which of the following could be the value of $\frac{n}{25}$?

- (A) 22
 (B) 23
 (C) 24
 (D) 25
 (E) 26

162. A fruit-salad mixture consists of apples, peaches, and grapes in the ratio 6:5:2, respectively, by weight. If 39 pounds of the mixture is prepared, the mixture includes how many more pounds of apples than grapes?

- (A) 15
 (B) 12
 (C) 9
 (D) 6
 (E) 4

163. This year Henry will save a certain amount of his income, and he will spend the rest. Next year Henry will have no income, but for each dollar that he saves this year, he will have $1+r$ dollars available to spend. In terms of r , what fraction of his income should Henry save this year so that next year the amount he has available to spend will be equal to half the amount that he spends this year?

- (A) $\frac{1}{r+2}$
 (B) $\frac{1}{2r+2}$
 (C) $\frac{1}{3r+2}$
 (D) $\frac{1}{r+3}$
 (E) $\frac{1}{2r+3}$

164. If $m^{-1} = -\frac{1}{3}$, then m^{-2} is equal to

- (A) -9
 (B) -3
 (C) $-\frac{1}{9}$
 (D) $\frac{1}{9}$
 (E) 9

165. Lois has x dollars more than Jim has, and together they have a total of y dollars. Which of the following represents the number of dollars that Jim has?
- (A) $\frac{y-x}{2}$
(B) $y - \frac{x}{2}$
(C) $\frac{y}{2} - x$
(D) $2y - x$
(E) $y - 2x$
166. During a certain season, a team won 80 percent of its first 100 games and 50 percent of its remaining games. If the team won 70 percent of its games for the entire season, what was the total number of games that the team played?
- (A) 180
(B) 170
(C) 156
(D) 150
(E) 105
167. Of 30 applicants for a job, 14 had at least 4 years' experience, 18 had degrees, and 3 had less than 4 years' experience and did not have a degree. How many of the applicants had at least 4 years' experience and a degree?
- (A) 14
(B) 13
(C) 9
(D) 7
(E) 5
168. If $1 + \frac{1}{x} = 2 - \frac{2}{x}$, then $x =$
- (A) -1
(B) $\frac{1}{3}$
(C) $\frac{2}{3}$
(D) 2
(E) 3
169. Last year, for every 100 million vehicles that traveled on a certain highway, 96 vehicles were involved in accidents. If 3 billion vehicles traveled on the highway last year, how many of those vehicles were involved in accidents? (1 billion = 1,000,000,000)
- (A) 288
(B) 320
(C) 2,880
(D) 3,200
(E) 28,800
170. Thirty percent of the members of a swim club have passed the lifesaving test. Among the members who have *not* passed the test, 12 have taken the preparatory course and 30 have not taken the course. How many members are there in the swim club?
- (A) 60
(B) 80
(C) 100
(D) 120
(E) 140
171. What is the difference between the sixth and the fifth terms of the sequence 2, 4, 7, ... whose n th term is $n + 2^{n-1}$?
- (A) 2
(B) 3
(C) 6
(D) 16
(E) 17
172. If $(x-1)^2 = 400$, which of the following could be the value of $x-5$?
- (A) 15
(B) 14
(C) -24
(D) -25
(E) -26
173. Which of the following describes all values of x for which $1 - x^2 \geq 0$?
- (A) $x \geq 1$
(B) $x \leq -1$
(C) $0 \leq x \leq 1$
(D) $x \leq -1$ or $x \geq 1$
(E) $-1 \leq x \leq 1$

174. The probability is $\frac{1}{2}$ that a certain coin will turn up heads on any given toss. If the coin is to be tossed three times, what is the probability that on at least one of the tosses the coin will turn up tails?

- (A) $\frac{1}{8}$
 (B) $\frac{1}{2}$
 (C) $\frac{3}{4}$
 (D) $\frac{7}{8}$
 (E) $\frac{15}{16}$

175. Of the final grades received by the students in a certain math course, $\frac{1}{5}$ are A's, $\frac{1}{4}$ are B's, $\frac{1}{2}$ are C's, and the remaining 10 grades are D's. What is the number of students in the course?

- (A) 80
 (B) 110
 (C) 160
 (D) 200
 (E) 400

176. As x increases from 165 to 166, which of the following must increase?

I. $2x - 5$

II. $1 - \frac{1}{x}$

III. $\frac{1}{x^2 - x}$

- (A) I only
 (B) III only
 (C) I and II
 (D) I and III
 (E) II and III

177. A rectangular box is 10 inches wide, 10 inches long, and 5 inches high. What is the greatest possible (straight-line) distance, in inches, between any two points on the box?

- (A) 15
 (B) 20
 (C) 25
 (D) $10\sqrt{2}$
 (E) $10\sqrt{3}$

Club	Number of Students
Chess	40
Drama	30
Math	25

178. The table above shows the number of students in three clubs at McAuliffe School. Although no student is in all three clubs, 10 students are in both Chess and Drama, 5 students are in both Chess and Math, and 6 students are in both Drama and Math. How many different students are in the three clubs?

- (A) 68
 (B) 69
 (C) 74
 (D) 79
 (E) 84

179. The ratio of two quantities is 3 to 4. If each of the quantities is increased by 5, what is the ratio of these two new quantities?

- (A) $\frac{3}{4}$
 (B) $\frac{8}{9}$
 (C) $\frac{18}{19}$
 (D) $\frac{23}{24}$

- (E) It cannot be determined from the information given.

180. If the average (arithmetic mean) of x and y is 60 and the average (arithmetic mean) of y and z is 80, what is the value of $z - x$?
- (A) 70
(B) 40
(C) 20
(D) 10
(E) It cannot be determined from the information given.
181. If $\frac{1}{2}$ of the air in a tank is removed with each stroke of a vacuum pump, what fraction of the original amount of air has been removed after 4 strokes?
- (A) $\frac{15}{16}$
(B) $\frac{7}{8}$
(C) $\frac{1}{4}$
(D) $\frac{1}{8}$
(E) $\frac{1}{16}$
182. If the two-digit integers M and N are positive and have the same digits, but in reverse order, which of the following CANNOT be the sum of M and N ?
- (A) 181
(B) 165
(C) 121
(D) 99
(E) 44
183. Car X and Car Y traveled the same 80-mile route. If Car X took 2 hours and Car Y traveled at an average speed that was 50 percent faster than the average speed of Car X, how many hours did it take Car Y to travel the route?
- (A) $\frac{2}{3}$
(B) 1
(C) $1\frac{1}{3}$
(D) $1\frac{3}{5}$
(E) 3
184. If the average (arithmetic mean) of the four numbers K , $2K + 3$, $3K - 5$, and $5K + 1$ is 63, what is the value of K ?
- (A) 11
(B) $15\frac{3}{4}$
(C) 22
(D) 23
(E) $25\frac{3}{10}$
185. If p is an even integer and q is an odd integer, which of the following must be an odd integer?
- (A) $\frac{p}{q}$
(B) pq
(C) $2p + q$
(D) $2(p + q)$
(E) $\frac{3p}{q}$
186. Drum X is $\frac{1}{2}$ full of oil and Drum Y, which has twice the capacity of Drum X, is $\frac{2}{3}$ full of oil. If all of the oil in Drum X is poured into Drum Y, then Drum Y will be filled to what fraction of its capacity?
- (A) $\frac{3}{4}$
(B) $\frac{5}{6}$
(C) $\frac{11}{12}$
(D) $\frac{7}{6}$
(E) $\frac{11}{6}$

187. If $x > 0$, $\frac{x}{50} + \frac{x}{25}$ is what percent of x ?

- (A) 6%
- (B) 25%
- (C) 37%
- (D) 60%
- (E) 75%

188. If the operation \odot is defined for all a and b by the equation $a \odot b = \frac{a^2b}{3}$, then $2 \odot (3 \odot -1) =$

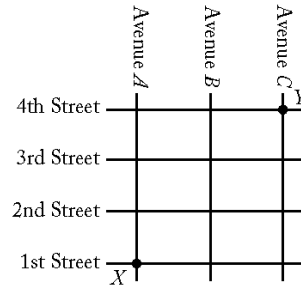
- (A) 4
- (B) 2
- (C) $-\frac{4}{3}$
- (D) -2
- (E) -4

189. The inside dimensions of a rectangular wooden box are 6 inches by 8 inches by 10 inches. A cylindrical canister is to be placed inside the box so that it stands upright when the closed box rests on one of its six faces. Of all such canisters that could be used, what is the radius, in inches, of the one that has maximum volume?

- (A) 3
- (B) 4
- (C) 5
- (D) 6
- (E) 8

190. What is the units digit of $(13)^4(17)^2(29)^3$?

- (A) 9
- (B) 7
- (C) 5
- (D) 3
- (E) 1



191. Pat will walk from Intersection X to Intersection Y along a route that is confined to the square grid of four streets and three avenues shown in the map above. How many routes from X to Y can Pat take that have the minimum possible length?

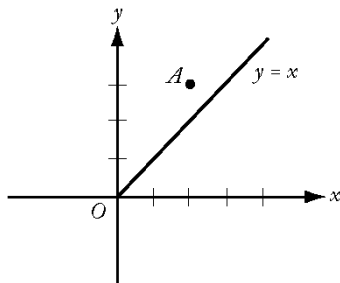
- (A) 6
- (B) 8
- (C) 10
- (D) 14
- (E) 16

192. The ratio, by volume, of soap to alcohol to water in a certain solution is 2:50:100. The solution will be altered so that the ratio of soap to alcohol is doubled while the ratio of soap to water is halved. If the altered solution will contain 100 cubic centimeters of alcohol, how many cubic centimeters of water will it contain?

- (A) 50
- (B) 200
- (C) 400
- (D) 625
- (E) 800

193. If 75 percent of a class answered the first question on a certain test correctly, 55 percent answered the second question on the test correctly, and 20 percent answered neither of the questions correctly, what percent answered both correctly?

- (A) 10%
- (B) 20%
- (C) 30%
- (D) 50%
- (E) 65%



194. In the rectangular coordinate system above, the line $y = x$ is the perpendicular bisector of segment AB (not shown), and the x -axis is the perpendicular bisector of segment BC (not shown). If the coordinates of point A are $(2,3)$, what are the coordinates of point C ?
- (A) $(-3,-2)$
 (B) $(-3,2)$
 (C) $(2,-3)$
 (D) $(3,-2)$
 (E) $(2,3)$
195. A store currently charges the same price for each towel that it sells. If the current price of each towel were to be increased by \$1, 10 fewer of the towels could be bought for \$120, excluding sales tax. What is the current price of each towel?
- (A) \$ 1
 (B) \$ 2
 (C) \$ 3
 (D) \$ 4
 (E) \$12

Number of Solid-Colored Marbles in Three Jars			
Jar	Number of red marbles	Number of green marbles	Total number of red and green marbles
P	x	y	80
Q	y	z	120
R	x	z	160

196. In the table above, what is the number of green marbles in Jar R?

- (A) 70
 (B) 80
 (C) 90
 (D) 100
 (E) 110

197. A point on the edge of a fan blade that is rotating in a plane is 10 centimeters from the center of the fan. What is the distance traveled, in centimeters, by this point in 15 seconds when the fan runs at the rate of 300 revolutions per minute?
- (A) 750π
 (B) $1,500\pi$
 (C) $1,875\pi$
 (D) $3,000\pi$
 (E) $7,500\pi$
198. If $n = 4p$, where p is a prime number greater than 2, how many different positive even divisors does n have, including n ?
- (A) Two
 (B) Three
 (C) Four
 (D) Six
 (E) Eight
- I. 72, 73, 74, 75, 76
 II. 74, 74, 74, 74, 74
 III. 62, 74, 74, 74, 89

199. The data sets I, II, and III above are ordered from greatest standard deviation to least standard deviation in which of the following?
- (A) I, II, III
 (B) I, III, II
 (C) II, III, I
 (D) III, I, II
 (E) III, II, I

200. Of the 50 researchers in a workgroup, 40 percent will be assigned to Team A and the remaining 60 percent to Team B. However, 70 percent of the researchers prefer Team A and 30 percent prefer Team B. What is the lowest possible number of researchers who will NOT be assigned to the team they prefer?

- (A) 15
- (B) 17
- (C) 20
- (D) 25
- (E) 30

201. If m is the average (arithmetic mean) of the first 10 positive multiples of 5 and if M is the median of the first 10 positive multiples of 5, what is the value of $M - m$?

- (A) -5
- (B) 0
- (C) 5
- (D) 25
- (E) 27.5

202. If $m > 0$ and x is m percent of y , then, in terms of m , y is what percent of x ?

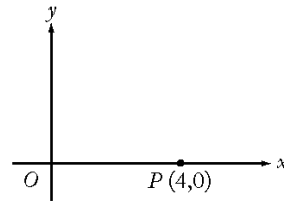
- (A) $100m$
- (B) $\frac{1}{100m}$
- (C) $\frac{1}{m}$
- (D) $\frac{10}{m}$
- (E) $\frac{10,000}{m}$

203. What is the 25th digit to the right of the decimal point in the decimal form of $\frac{6}{11}$?

- (A) 3
- (B) 4
- (C) 5
- (D) 6
- (E) 7

204. John and Mary were each paid x dollars in advance to do a certain job together. John worked on the job for 10 hours and Mary worked 2 hours less than John. If Mary gave John y dollars of her payment so that they would have received the same hourly wage, what was the dollar amount, in terms of y , that John was paid in advance?

- (A) $4y$
- (B) $5y$
- (C) $6y$
- (D) $8y$
- (E) $9y$



205. In the rectangular coordinate system above, if point R (not shown) lies on the positive y -axis and the area of triangle ORP is 12, what is the y -coordinate of point R ?

- (A) 3
- (B) 6
- (C) 9
- (D) 12
- (E) 24

206. Car A is 20 miles behind Car B, which is traveling in the same direction along the same route as Car A. Car A is traveling at a constant speed of 58 miles per hour and Car B is traveling at a constant speed of 50 miles per hour. How many hours will it take for Car A to overtake and drive 8 miles ahead of Car B?

- (A) 1.5
- (B) 2.0
- (C) 2.5
- (D) 3.0
- (E) 3.5

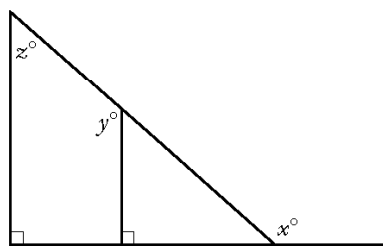
207. For the past n days, the average (arithmetic mean) daily production at a company was 50 units. If today's production of 90 units raises the average to 55 units per day, what is the value of n ?

- (A) 30
(B) 18
(C) 10
(D) 9
(E) 7

$$\left(\frac{x+1}{x-1}\right)^2$$

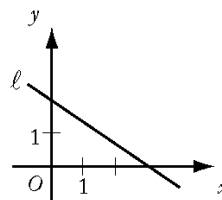
208. If $x \neq 0$ and $x \neq 1$, and if x is replaced by $\frac{1}{x}$ everywhere in the expression above, then the resulting expression is equivalent to

- (A) $\left(\frac{x+1}{x-1}\right)^2$
(B) $\left(\frac{x-1}{x+1}\right)^2$
(C) $\frac{x^2+1}{1-x^2}$
(D) $\frac{x^2-1}{x^2+1}$
(E) $-\left(\frac{x-1}{x+1}\right)^2$



209. In the figure above, if $z = 50$, then $x + y =$

- (A) 230
(B) 250
(C) 260
(D) 270
(E) 290

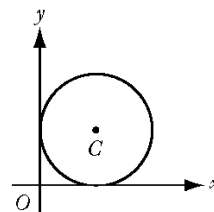


210. In the coordinate system above, which of the following is the equation of line l ?

- (A) $2x - 3y = 6$
(B) $2x + 3y = 6$
(C) $3x + 2y = 6$
(D) $2x - 3y = -6$
(E) $3x - 2y = -6$

211. If a two-digit positive integer has its digits reversed, the resulting integer differs from the original by 27. By how much do the two digits differ?

- (A) 3
(B) 4
(C) 5
(D) 6
(E) 7



212. The circle with center C shown above is tangent to both axes. If the distance from O to C is equal to k , what is the radius of the circle, in terms of k ?

- (A) k
(B) $\frac{k}{\sqrt{2}}$
(C) $\frac{k}{\sqrt{3}}$
(D) $\frac{k}{2}$
(E) $\frac{k}{3}$

213. In an electric circuit, two resistors with resistances x and y are connected in parallel. In this case, if r is the combined resistance of these two resistors, then the reciprocal of r is equal to the sum of the reciprocals of x and y . What is r in terms of x and y ?
- (A) xy
(B) $x + y$
(C) $\frac{1}{x + y}$
(D) $\frac{xy}{x + y}$
(E) $\frac{x + y}{xy}$
214. Xavier, Yvonne, and Zelda each try independently to solve a problem. If their individual probabilities for success are $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{5}{8}$, respectively, what is the probability that Xavier and Yvonne, but not Zelda, will solve the problem?
- (A) $\frac{11}{8}$
(B) $\frac{7}{8}$
(C) $\frac{9}{64}$
(D) $\frac{5}{64}$
(E) $\frac{3}{64}$
215. If $\frac{1}{x} - \frac{1}{x+1} = \frac{1}{x+4}$, then x could be
- (A) 0
(B) -1
(C) -2
(D) -3
(E) -4
216. $\left(\frac{1}{2}\right)^{-3} \left(\frac{1}{4}\right)^{-2} \left(\frac{1}{16}\right)^{-1} =$
- (A) $\left(\frac{1}{2}\right)^{-48}$
(B) $\left(\frac{1}{2}\right)^{-11}$
(C) $\left(\frac{1}{2}\right)^{-6}$
(D) $\left(\frac{1}{8}\right)^{-11}$
(E) $\left(\frac{1}{8}\right)^{-6}$
217. In a certain game, a large container is filled with red, yellow, green, and blue beads worth, respectively, 7, 5, 3, and 2 points each. A number of beads are then removed from the container. If the product of the point values of the removed beads is 147,000, how many red beads were removed?
- (A) 5
(B) 4
(C) 3
(D) 2
(E) 0
218. If $\frac{2}{1+\frac{2}{y}} = 1$, then $y =$
- (A) -2
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 2
(E) 3

219. If a , b , and c are consecutive positive integers and $a < b < c$, which of the following must be true?
- $c - a = 2$
 - abc is an even integer.
 - $\frac{a + b + c}{3}$ is an integer.
- (A) I only
 (B) II only
 (C) I and II only
 (D) II and III only
 (E) I, II, and III
220. A part-time employee whose hourly wage was increased by 25 percent decided to reduce the number of hours worked per week so that the employee's total weekly income would remain unchanged. By what percent should the number of hours worked be reduced?
- (A) 12.5%
 (B) 20%
 (C) 25%
 (D) 50%
 (E) 75%
221. Of the 200 students at College T majoring in one or more of the sciences, 130 are majoring in chemistry and 150 are majoring in biology. If at least 30 of the students are not majoring in either chemistry or biology, then the number of students majoring in *both* chemistry and biology could be any number from
- (A) 20 to 50
 (B) 40 to 70
 (C) 50 to 130
 (D) 110 to 130
 (E) 110 to 150
222. If $5 - \frac{6}{x} = x$, then x has how many possible values?
- (A) None
 (B) One
 (C) Two
 (D) A finite number greater than two
 (E) An infinite number
223. Seed mixture X is 40 percent ryegrass and 60 percent bluegrass by weight; seed mixture Y is 25 percent ryegrass and 75 percent fescue. If a mixture of X and Y contains 30 percent ryegrass, what percent of the weight of the mixture is X?
- (A) 10%
 (B) $33\frac{1}{3}\%$
 (C) 40%
 (D) 50%
 (E) $66\frac{2}{3}\%$
224. If n is a positive integer, then $n(n + 1)(n + 2)$ is
- (A) even only when n is even
 (B) even only when n is odd
 (C) odd whenever n is odd
 (D) divisible by 3 only when n is odd
 (E) divisible by 4 whenever n is even
225. A straight pipe 1 yard in length was marked off in fourths and also in thirds. If the pipe was then cut into separate pieces at each of these markings, which of the following gives all the different lengths of the pieces, in fractions of a yard?
- (A) $\frac{1}{6}$ and $\frac{1}{4}$ only
 (B) $\frac{1}{4}$ and $\frac{1}{3}$ only
 (C) $\frac{1}{6}$, $\frac{1}{4}$, and $\frac{1}{3}$
 (D) $\frac{1}{12}$, $\frac{1}{6}$, and $\frac{1}{4}$
 (E) $\frac{1}{12}$, $\frac{1}{6}$, and $\frac{1}{3}$
226. If $\frac{0.0015 \times 10^m}{0.03 \times 10^k} = 5 \times 10^7$, then $m - k =$
- (A) 9
 (B) 8
 (C) 7
 (D) 6
 (E) 5

227. If $x + y = a$ and $x - y = b$, then $2xy =$

- (A) $\frac{a^2 - b^2}{2}$
(B) $\frac{b^2 - a^2}{2}$
(C) $\frac{a - b}{2}$
(D) $\frac{ab}{2}$
(E) $\frac{a^2 + b^2}{2}$

p, r, s, t, u

228. An arithmetic sequence is a sequence in which each term after the first is equal to the sum of the preceding term and a constant. If the list of letters shown above is an arithmetic sequence, which of the following must also be an arithmetic sequence?

- I. $2p, 2r, 2s, 2t, 2u$
II. $p - 3, r - 3, s - 3, t - 3, u - 3$
III. p^2, r^2, s^2, t^2, u^2

- (A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

229. Right triangle PQR is to be constructed in the xy -plane so that the right angle is at P and \overline{PR} is parallel to the x -axis. The x - and y -coordinates of $P, Q,$ and R are to be integers that satisfy the inequalities $-4 \leq x \leq 5$ and $6 \leq y \leq 16$. How many different triangles with these properties could be constructed?

- (A) 110
(B) 1,100
(C) 9,900
(D) 10,000
(E) 12,100

230. The value of $\frac{2^{-14} + 2^{-15} + 2^{-16} + 2^{-17}}{5}$ is how many times the value of 2^{-17} ?

- (A) $\frac{3}{2}$
(B) $\frac{5}{2}$
(C) 3
(D) 4
(E) 5

参考答案

1. A	31. B	61. E	91. B
2. B	32. B	62. A	92. C
3. C	33. C	63. A	93. B
4. E	34. A	64. B	94. B
5. C	35. B	65. B	95. E
6. C	36. A	66. E	96. D
7. B	37. A	67. B	97. B
8. E	38. B	68. D	98. C
9. E	39. B	69. C	99. B
10. D	40. C	70. C	100. D
11. E	41. A	71. A	101. D
12. A	42. E	72. B	102. B
13. C	43. B	73. D	103. E
14. A	44. D	74. D	104. B
15. B	45. B	75. E	105. C
16. D	46. C	76. D	106. B
17. D	47. C	77. D	107. D
18. E	48. C	78. C	108. B
19. E	49. B	79. A	109. C
20. A	50. B	80. B	110. C
21. B	51. C	81. A	111. B
22. A	52. E	82. A	112. D
23. B	53. E	83. C	113. D
24. D	54. A	84. A	114. B
25. E	55. E	85. A	115. C
26. B	56. B	86. A	116. B
27. E	57. A	87. E	117. E
28. B	58. B	88. D	118. C
29. C	59. D	89. D	119. D
30. B	60. D	90. E	120. C

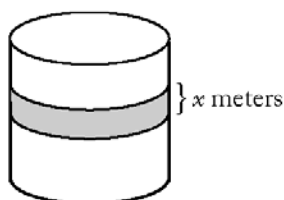
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- | | | | |
|--------|--------|--------|--------|
| 121. C | 151. B | 181. A | 211. A |
| 122. E | 152. E | 182. A | 212. B |
| 123. B | 153. D | 183. C | 213. D |
| 124. B | 154. B | 184. D | 214. E |
| 125. E | 155. D | 185. C | 215. C |
| 126. C | 156. D | 186. C | 216. B |
| 127. A | 157. B | 187. A | 217. D |
| 128. A | 158. E | 188. E | 218. D |
| 129. D | 159. D | 189. B | 219. E |
| 130. E | 160. D | 190. E | 220. B |
| 131. E | 161. E | 191. C | 221. D |
| 132. D | 162. B | 192. E | 222. C |
| 133. E | 163. E | 193. D | 223. B |
| 134. B | 164. D | 194. D | 224. E |
| 135. C | 165. A | 195. C | 225. D |
| 136. A | 166. D | 196. D | 226. A |
| 137. E | 167. E | 197. B | 227. A |
| 138. E | 168. E | 198. C | 228. D |
| 139. B | 169. C | 199. D | 229. C |
| 140. D | 170. A | 200. A | 230. C |
| 141. C | 171. E | 201. B | |
| 142. E | 172. C | 202. E | |
| 143. A | 173. E | 203. C | |
| 144. C | 174. D | 204. E | |
| 145. C | 175. D | 205. B | |
| 146. C | 176. C | 206. E | |
| 147. A | 177. A | 207. E | |
| 148. D | 178. C | 208. A | |
| 149. E | 179. E | 209. D | |
| 150. A | 180. B | 210. B | |

第 9 章 Data Sufficiency 练习题

- What is the value of $|x|$?
 - $x = -|x|$
 - $x^2 = 4$
- What percent of a group of people are women with red hair?
 - Of the women in the group, 5 percent have red hair.
 - Of the men in the group, 10 percent have red hair.
- In a certain class, one student is to be selected at random to read. What is the probability that a boy will read?
 - Two-thirds of the students in the class are boys.
 - Ten of the students in the class are girls.
- In College X the number of students enrolled in both a chemistry course and a biology course is how much less than the number of students enrolled in neither?
 - In College X there are 60 students enrolled in a chemistry course.
 - In College X there are 85 students enrolled in a biology course.
- A certain expressway has Exits J, K, L, and M, in that order. What is the road distance from Exit K to Exit L?
 - The road distance from Exit J to Exit L is 21 kilometers.
 - The road distance from Exit K to Exit M is 26 kilometers.
- If n is an integer, is $n + 1$ odd?
 - $n + 2$ is an even integer.
 - $n - 1$ is an odd integer.
- For which type of investment, J or K, is the annual rate of return greater?
 - Type J returns \$115 per \$1,000 invested for any one-year period and type K returns \$300 per \$2,500 invested for any one-year period.
 - The annual rate of return for an investment of type K is 12 percent.
- A citrus fruit grower receives \$15 for each crate of oranges shipped and \$18 for each crate of grapefruit shipped. How many crates of oranges did the grower ship last week?
 - Last week the number of crates of oranges that the grower shipped was 20 more than twice the number of crates of grapefruit shipped.
 - Last week the grower received a total of \$38,700 from the crates of oranges and grapefruit shipped.
- If Pat saved \$600 of his earnings last month, how much did Pat earn last month?
 - Pat spent $\frac{1}{2}$ of his earnings last month for living expenses and saved $\frac{1}{3}$ of the remainder.
 - Of his earnings last month, Pat paid twice as much in taxes as he saved.
- Water is pumped into a partially filled tank at a constant rate through an inlet pipe. At the same time, water is pumped out of the tank at a constant rate through an outlet pipe. At what rate, in gallons per minute, is the amount of water in the tank increasing?
 - The amount of water initially in the tank is 200 gallons.
 - Water is pumped into the tank at a rate of 10 gallons per minute and out of the tank at a rate of 10 gallons every $2\frac{1}{2}$ minutes.
- Is x a negative number?
 - $9x > 10x$
 - $x + 3$ is positive.
- If i and j are integers, is $i + j$ an even integer?
 - $i < 10$
 - $i = j$
- The charge for a telephone call between City R and City S is \$0.42 for each of the first 3 minutes and \$0.18 for each additional minute. A certain call between these two cities lasted for x minutes, where x is an integer. How many minutes long was the call?
 - The charge for the first 3 minutes of the call was \$0.36 less than the charge for the remainder of the call.
 - The total charge for the call was \$2.88.

14. If Car X followed Car Y across a certain bridge that is $\frac{1}{2}$ mile long, how many seconds did it take Car X to travel across the bridge?
- (1) Car X drove onto the bridge exactly 3 seconds after Car Y drove onto the bridge and drove off the bridge exactly 2 seconds after Car Y drove off the bridge.
 - (2) Car Y traveled across the bridge at a constant speed of 30 miles per hour.
15. If $n + k = m$, what is the value of k ?
- (1) $n = 10$
 - (2) $m + 10 = n$
16. Is x an integer?
- (1) $\frac{x}{2}$ is an integer.
 - (2) $2x$ is an integer.
17. Is the integer P odd?
- (1) The sum of P , $P + 4$, and $P + 11$ is even.
 - (2) The sum of $P - 3$, P , and $P + 11$ is odd.
18. What is the maximum number of rectangular blocks, each with dimensions 12 centimeters by 6 centimeters by 4 centimeters, that will fit inside rectangular Box X?
- (1) When Box X is filled with the blocks and rests on a certain side, there are 25 blocks in the bottom layer.
 - (2) The inside dimensions of Box X are 60 centimeters by 30 centimeters by 20 centimeters.
19. If sequence S has 200 terms, what is the 192nd term of S ?
- (1) The first term of S is -40 .
 - (2) Each term of S after the first term is 3 less than the preceding term.
20. In $\triangle PQR$, if $PQ = x$, $QR = x + 2$, and $PR = y$, which of the three angles of $\triangle PQR$ has the greatest degree measure?
- (1) $y = x + 3$
 - (2) $x = 2$
21. What percent of the drama club members enrolled at a certain school are female students?
- (1) Of the female students enrolled at the school, 40 percent are members of the drama club.
 - (2) Of the male students enrolled at the school, 25 percent are members of the drama club.
22. A family-size box of cereal contains more cereal and costs more than the regular-size box of cereal. What is the cost per ounce of the family-size box of cereal?
- (1) The family-size box of cereal contains 10 ounces more than the regular-size box of cereal.
 - (2) The family-size box of cereal costs \$5.40.
23. The profit from the sale of a certain appliance increases, though not proportionally, with the number of units sold. Did the profit exceed \$4 million on sales of 380,000 units?
- (1) The profit exceeded \$2 million on sales of 200,000 units.
 - (2) The profit exceeded \$5 million on sales of 350,000 units.
24. If n is an integer, is n even?
- (1) $n^2 - 1$ is an odd integer.
 - (2) $3n + 4$ is an even integer.
25. Carmen currently works 30 hours per week at her part-time job. If her gross hourly wage were to increase by \$1.50, how many fewer hours could she work per week and still earn the same gross weekly pay as before the increase?
- (1) Her gross weekly pay is currently \$225.00.
 - (2) An increase of \$1.50 would represent an increase of 20 percent of her current gross hourly wage.
26. The number n of units of its product that Company X is scheduled to produce in month t of its next fiscal year is given by the formula $n = \frac{900}{1 + c2^{-t}}$, where c is a constant and t is a positive integer between 1 and 6, inclusive. What is the number of units of its product that Company X is scheduled to produce in month 6 of its next fiscal year?
- (1) Company X is scheduled to produce 180 units of its product in month 1 of its next fiscal year.
 - (2) Company X is scheduled to produce 300 units of its product in month 2 of its next fiscal year.

27. When 200 gallons of oil were removed from a tank, the volume of oil left in the tank was $\frac{3}{7}$ of the tank's capacity. What was the tank's capacity?
- (1) Before the 200 gallons were removed, the volume of oil in the tank was $\frac{1}{2}$ of the tank's capacity.
 - (2) After the 200 gallons were removed, the volume of the oil left in the tank was 1,600 gallons less than the tank's capacity.
28. Division R of Company Q has 1,000 employees. What is the average (arithmetic mean) annual salary of the employees at Company Q?
- (1) The average annual salary of the employees in Division R is \$30,000.
 - (2) The average annual salary of the employees at Company Q who are not in Division R is \$35,000.

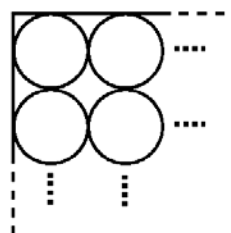


29. A circular tub has a band painted around its circumference, as shown above. What is the surface area of this painted band?
- (1) $x = 0.5$
 - (2) The height of the tub is 1 meter.
30. What is the value of integer n ?
- (1) $n(n + 1) = 6$
 - (2) $2^{2n} = 16$

$$d = 0.43t7$$

31. If t denotes the thousandths digit in the decimal representation of d above, what digit is t ?
- (1) If d were rounded to the nearest hundredth, the result would be 0.44.
 - (2) If d were rounded to the nearest thousandth, the result would be 0.436.

32. Jerry bought 7 clothing items, including a coat, and the sum of the prices of these items was \$365. If there was no sales tax on any clothing item with a price of less than \$100 and a 7 percent sales tax on all other clothing items, what was the total sales tax on the 7 items that Jerry bought?
- (1) The price of the coat was \$125.
 - (2) The average (arithmetic mean) price for the 6 items other than the coat was \$40.
33. What was the price at which a merchant sold a certain appliance?
- (1) The merchant's gross profit on the appliance was 20 percent of the price at which the merchant sold the appliance.
 - (2) The price at which the merchant sold the appliance was \$50 more than the merchant's cost of the appliance.



34. The inside of a rectangular carton is 48 centimeters long, 32 centimeters wide, and 15 centimeters high. The carton is filled to capacity with k identical cylindrical cans of fruit that stand upright in rows and columns, as indicated in the figure above. If the cans are 15 centimeters high, what is the value of k ?
- (1) Each of the cans has a radius of 4 centimeters.
 - (2) Six of the cans fit exactly along the length of the carton.

$$\begin{cases} x - 4 = z \\ y - x = 8 \\ 8 - z = t \end{cases}$$

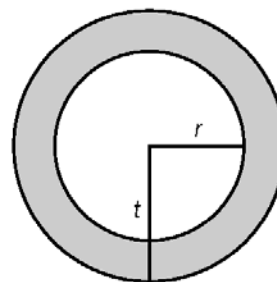
35. For the system of equations given, what is the value of z ?
- (1) $x = 7$
 - (2) $t = 5$

36. For all integers n , the function f is defined by $f(n) = a^n$, where a is a constant. What is the value of $f(1)$?
- (1) $f(2) = 100$
 (2) $f(3) = -1,000$
37. The selling price of an article is equal to the cost of the article plus the markup. The markup on a certain television set is what percent of the selling price?
- (1) The markup on the television set is 25 percent of the cost.
 (2) The selling price of the television set is \$250.
38. If p_1 and p_2 are the populations and r_1 and r_2 are the numbers of representatives of District 1 and District 2, respectively, the ratio of the population to the number of representatives is greater for which of the two districts?
- (1) $p_1 > p_2$
 (2) $r_2 > r_1$
39. In a random sample of 80 adults, how many are college graduates?
- (1) In the sample, the number of adults who are not college graduates is 3 times the number who are college graduates.
 (2) In the sample, the number of adults who are not college graduates is 40 more than the number who are college graduates.

	R	S	T	U
R	0	y	x	62
S	y	0	56	75
T	x	56	0	69
U	62	75	69	0

40. The table above shows the distance, in kilometers, by the most direct route, between any two of the four cities, R, S, T, and U. For example, the distance between City R and City U is 62 kilometers. What is the value of x ?
- (1) By the most direct route, the distance between S and T is twice the distance between S and R.
 (2) By the most direct route, the distance between T and U is 1.5 times the distance between R and T.

41. What is the value of the two-digit integer x ?
- (1) The sum of the two digits is 3.
 (2) x is divisible by 3.

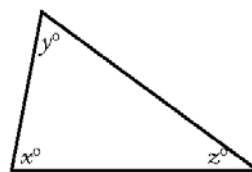


42. The figure above shows the circular cross section of a concrete water pipe. If the inside radius of the pipe is r feet and the outside radius of the pipe is t feet, what is the value of r ?
- (1) The ratio of $t - r$ to r is 0.15 and $t - r$ is equal to 0.3 foot.
 (2) The area of the concrete in the cross section is 1.29π square feet.
43. What is the tenths digit in the decimal representation of a certain number?
- (1) The number is less than $\frac{1}{3}$.
 (2) The number is greater than $\frac{1}{4}$.
44. Robots X, Y, and Z each assemble components at their respective constant rates. If r_x is the ratio of Robot X's constant rate to Robot Z's constant rate and r_y is the ratio of Robot Y's constant rate to Robot Z's constant rate, is Robot Z's constant rate the greatest of the three?
- (1) $r_x < r_y$
 (2) $r_y < 1$
45. If r is a constant and $a_n = m$ for all positive integers n , for how many values of n is $a_n < 100$?
- (1) $a_{50} = 500$
 (2) $a_{100} + a_{105} = 2,050$

46. If r is represented by the decimal $0.t5$, what is the digit t ?
- (1) $r < \frac{1}{3}$
 - (2) $r < \frac{1}{10}$
47. If the two floors in a certain building are 9 feet apart, how many steps are there in a set of stairs that extends from the first floor to the second floor of the building?
- (1) Each step is $\frac{3}{4}$ foot high.
 - (2) Each step is 1 foot wide.
48. In June 1989, what was the ratio of the number of sales transactions made by Salesperson X to the number of sales transactions made by Salesperson Y?
- (1) In June 1989, Salesperson X made 50 percent more sales transactions than Salesperson Y did in May 1989.
 - (2) In June 1989, Salesperson Y made 25 percent more sales transactions than in May 1989.
49. If $a < x < b$ and $c < y < d$, is $x < y$?
- (1) $a < c$
 - (2) $b < d$
50. How many people are directors of both Company K and Company R?
- (1) There were 17 directors present at a joint meeting of the directors of Company K and Company R, and no directors were absent.
 - (2) Company K has 12 directors and Company R has 8 directors.
51. If x and y are positive, is $\frac{x}{y}$ greater than 1?
- (1) $xy > 1$
 - (2) $x - y > 0$
52. A clothing store acquired an item at a cost of x dollars and sold the item for y dollars. The store's gross profit from the item was what percent of its cost for the item?
- (1) $y - x = 20$
 - (2) $\frac{y}{x} = \frac{5}{4}$

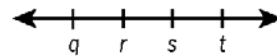
$$(n - x) + (n - y) + (n - z) + (n - k)$$

53. What is the value of the expression above?
- (1) The average (arithmetic mean) of x , y , z , and k is n .
 - (2) x , y , z , and k are consecutive integers.
54. A taxi company charges f cents for the first mile of the taxi ride and m cents for each additional mile. How much does the company charge for a 10-mile taxi ride?
- (1) The company charges \$0.90 for a 2-mile ride.
 - (2) The company charges \$1.20 for a 4-mile ride.
55. Guy's net income equals his gross income minus his deductions. By what percent did Guy's net income change on January 1, 1989, when both his gross income and his deductions increased?
- (1) Guy's gross income increased by 4 percent on January 1, 1989.
 - (2) Guy's deductions increased by 15 percent on January 1, 1989.



56. What is the value of z in the triangle above?
- (1) $x + y = 139$
 - (2) $y + z = 108$
57. Max has \$125 consisting of bills each worth either \$5 or \$20. How many bills worth \$5 does Max have?
- (1) Max has fewer than 5 bills worth \$5 each.
 - (2) Max has more than 5 bills worth \$20 each.
58. If the ratio of the number of teachers to the number of students is the same in School District M and School District P, what is the ratio of the number of students in School District M to the number of students in School District P?
- (1) There are 10,000 more students in School District M than there are in School District P.
 - (2) The ratio of the number of teachers to the number of students in School District M is 1 to 20.

59. If a total of 84 students are enrolled in two sections of a calculus course, how many of the 84 students are female?
- (1) $\frac{2}{3}$ of the students in Section 1 are female.
 - (2) $\frac{1}{2}$ of the students in Section 2 are male.
60. What is the value of n in the equation $-25 + 19 + n = s$?
- (1) $s = 2$
 - (2) $\frac{n}{s} = 4$
61. At a certain picnic, each of the guests was served either a single scoop or a double scoop of ice cream. How many of the guests were served a double scoop of ice cream?
- (1) At the picnic, 60 percent of the guests were served a double scoop of ice cream.
 - (2) A total of 120 scoops of ice cream were served to all the guests at the picnic.
62. For a convention, a hotel charges a daily room rate of \$120 for 1 person and x dollars for each additional person. What is the charge for each additional person?
- (1) The daily cost per person for 4 people sharing the cost of a room equally is \$45.
 - (2) The daily cost per person for 2 people sharing the cost of a room equally is \$25 more than the corresponding cost for 4 people.
63. Stores L and M each sell a certain product at a different regular price. If both stores discount their regular price of the product, is the discount price at Store M less than the discount price at Store L?
- (1) At Store L the discount price is 10 percent less than the regular price; at Store M the discount price is 15 percent less than the regular price.
 - (2) At Store L the discount price is \$5 less than the regular store price; at Store M the discount price is \$6 less than the regular price.
64. If d denotes a decimal, is $d \geq 0.5$?
- (1) When d is rounded to the nearest tenth, the result is 0.5.
 - (2) When d is rounded to the nearest integer, the result is 1.
65. How many integers are there between, but not including, integers r and s ?
- (1) $s - r = 10$
 - (2) There are 9 integers between, but not including, $r + 1$ and $s + 1$.
66. If n and t are positive integers, is n a factor of t ?
- (1) $n = 3^{n-2}$
 - (2) $t = 3^n$
67. In a survey of 200 college graduates, 30 percent said they had received student loans during their college careers, and 40 percent said they had received scholarships. What percent of those surveyed said that they had received neither student loans nor scholarships during their college careers?
- (1) 25 percent of those surveyed said that they had received scholarships but no loans.
 - (2) 50 percent of those surveyed who said that they had received loans also said that they had received scholarships.
68. Three machines, K, M, and P, working simultaneously and independently at their respective constant rates, can complete a certain task in 24 minutes. How long does it take Machine K, working alone at its constant rate, to complete the task?
- (1) Machines M and P, working simultaneously and independently at their respective constant rates, can complete the task in 36 minutes.
 - (2) Machines K and P, working simultaneously and independently at their respective constant rates, can complete the task in 48 minutes.



69. Of the four numbers represented on the number line above, is r closest to zero?
- (1) $q = -s$
 - (2) $-t < q$

70. Mary persuaded n friends to donate \$500 each to her election campaign, and then each of these n friends persuaded n more people to donate \$500 each to Mary's campaign. If no one donated more than once and if there were no other donations, what was the value of n ?

- (1) The first n people donated $\frac{1}{16}$ of the total amount donated.
 (2) The total amount donated was \$120,000.

71. Carlotta can drive from her home to her office by one of two possible routes. If she must also return by one of these routes, what is the distance of the shorter route?

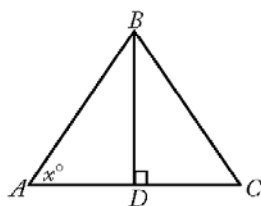
- (1) When she drives from her home to her office by the shorter route and returns by the longer route, she drives a total of 42 kilometers.
 (2) When she drives both ways, from her home to her office and back, by the longer route, she drives a total of 46 kilometers.

72. Is $x > y$?

- (1) $x = y + 2$
 (2) $\frac{x}{2} = y - 1$

73. If m is an integer, is m odd?

- (1) $\frac{m}{2}$ is not an even integer.
 (2) $m - 3$ is an even integer.



74. What is the area of triangular region ABC above?

- (1) The product of BD and AC is 20.
 (2) $x = 45$

75. In the xy -plane, the line with equation $ax + by + c = 0$, where $abc \neq 0$, has slope $\frac{2}{3}$. What is the value of b ?

- (1) $a = 4$
 (2) $c = -6$

76. If m , p , and t are positive integers and $m < p < t$, is the product mpt an even integer?

- (1) $t - p = p - m$
 (2) $t - m = 16$

77. Each week a certain salesman is paid a fixed amount equal to \$300, plus a commission equal to 5 percent of the amount of his sales that week over \$1,000. What is the total amount the salesman was paid last week?

- (1) The total amount the salesman was paid last week is equal to 10 percent of the amount of his sales last week.
 (2) The salesman's sales last week totaled \$5,000.

78. A total of \$60,000 was invested for one year. Part of this amount earned simple annual interest at the rate of x percent per year, and the rest earned simple annual interest at the rate of y percent per year. If the total interest earned by the \$60,000 for that year was \$4,080, what is the value of x ?

- (1) $x = \frac{3y}{4}$
 (2) The ratio of the amount that earned interest at the rate of x percent per year to the amount that earned interest at the rate of y percent per year was 3 to 2.

79. Leo can buy a certain computer for p_1 dollars in State A, where the sales tax is t_1 percent, or he can buy the same computer for p_2 dollars in State B, where the sales tax is t_2 percent. Is the total cost of the computer greater in State A than in State B?

- (1) $t_1 > t_2$
 (2) $p_1 t_1 > p_2 t_2$

80. If $r > 0$ and $s > 0$, is $\frac{r}{s} < \frac{s}{r}$?

- (1) $\frac{r}{3s} = \frac{1}{4}$
 (2) $s = r + 4$

$k, n, 12, 6, 17$

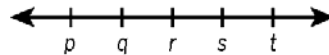
81. What is the value of n in the list above?

- (1) $k < n$
 (2) The median of the numbers in the list is 10.

82. If positive integer x is a multiple of 6 and positive integer y is a multiple of 14, is xy a multiple of 105?
- (1) x is a multiple of 9.
 - (2) y is a multiple of 25.
83. What is the value of $b + c$?
- (1) $ab + cd + ac + bd = 6$
 - (2) $a + d = 4$
84. What is the average (arithmetic mean) of j and k ?
- (1) The average (arithmetic mean) of $j + 2$ and $k + 4$ is 11.
 - (2) The average (arithmetic mean) of j , k , and 14 is 10.
85. Paula and Sandy were among those people who sold raffle tickets to raise money for Club X. If Paula and Sandy sold a total of 100 of the tickets, how many of the tickets did Paula sell?
- (1) Sandy sold $\frac{2}{3}$ as many of the raffle tickets as Paula did.
 - (2) Sandy sold 8 percent of all the raffle tickets sold for Club X.
86. A number of people each wrote down one of the first 30 positive integers. Were any of the integers written down by more than one of the people?
- (1) The number of people who wrote down an integer was greater than 40.
 - (2) The number of people who wrote down an integer was less than 70.
87. Is the number of seconds required to travel d_1 feet at r_1 feet per second greater than the number of seconds required to travel d_2 feet at r_2 feet per second?
- (1) d_1 is 30 greater than d_2 .
 - (2) r_1 is 30 greater than r_2 .
88. Last year, if Arturo spent a total of \$12,000 on his mortgage payments, real estate taxes, and home insurance, how much did he spend on his real estate taxes?
- (1) Last year, the total amount that Arturo spent on his real estate taxes and home insurance was $33\frac{1}{3}$ percent of the amount that he spent on his mortgage payments.
 - (2) Last year, the amount that Arturo spent on his real estate taxes was 20 percent of the total amount he spent on his mortgage payments and home insurance.
89. Is the number of members of Club X greater than the number of members of Club Y?
- (1) Of the members of Club X, 20 percent are also members of Club Y.
 - (2) Of the members of Club Y, 30 percent are also members of Club X.
90. If k , m , and t are positive integers and $\frac{k}{6} + \frac{m}{4} = \frac{t}{12}$, do t and 12 have a common factor greater than 1?
- (1) k is a multiple of 3.
 - (2) m is a multiple of 3.



91. In the figure above, is $CD > BC$?
- (1) $AD = 20$
 - (2) $AB = CD$
92. In a certain office, 50 percent of the employees are college graduates and 60 percent of the employees are over 40 years old. If 30 percent of those over 40 have master's degrees, how many of the employees over 40 have master's degrees?
- (1) Exactly 100 of the employees are college graduates.
 - (2) Of the employees 40 years old or less, 25 percent have master's degrees.



93. On the number line above, p , q , r , s , and t are five consecutive even integers in increasing order. What is the average (arithmetic mean) of these five integers?
- (1) $q + s = 24$
 - (2) The average (arithmetic mean) of q and r is 11.

94. If line k in the xy -plane has equation $y = mx + b$, where m and b are constants, what is the slope of k ?
- (1) k is parallel to the line with equation $y = (1 - m)x + b + 1$.
 - (2) k intersects the line with equation $y = 2x + 3$ at the point $(2, 7)$.
95. Is $rst = 1$?
- (1) $rs = 1$
 - (2) $st = 1$

TOTAL EXPENSES FOR THE
FIVE DIVISIONS OF COMPANY H



96. The figure above represents a circle graph of Company H's total expenses broken down by the expenses for each of its five divisions. If O is the center of the circle and if Company H's total expenses are \$5,400,000, what are the expenses for Division R?
- (1) $x = 94$
 - (2) The total expenses for Divisions S and T are twice as much as the expenses for Division R.
97. If x is negative, is $x < -3$?
- (1) $x^2 > 9$
 - (2) $x^3 < -9$
98. Seven different numbers are selected from the integers 1 to 100, and each number is divided by 7. What is the sum of the remainders?
- (1) The range of the seven remainders is 6.
 - (2) The seven numbers selected are consecutive integers.

r	s	t
u	v	w
x	y	z

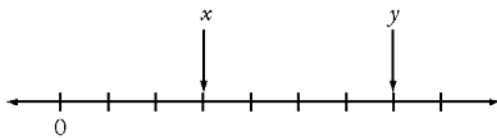
99. Each of the letters in the table above represents one of the numbers 1, 2, or 3, and each of these numbers occurs exactly once in each row and exactly once in each column. What is the value of r ?
- (1) $v + z = 6$
 - (2) $s + t + u + x = 6$
100. If $[x]$ denotes the greatest integer less than or equal to x , is $[x] = 0$?
- (1) $5x + 1 = 3 + 2x$
 - (2) $0 < x < 1$
101. Material A costs \$3 per kilogram, and Material B costs \$5 per kilogram. If 10 kilograms of Material K consists of x kilograms of Material A and y kilograms of Material B, is $x > y$?
- (1) $y > 4$
 - (2) The cost of the 10 kilograms of Material K is less than \$40.
102. While on a straight road, Car X and Car Y are traveling at different constant rates. If Car X is now 1 mile ahead of Car Y, how many minutes from now will Car X be 2 miles ahead of Car Y?
- (1) Car X is traveling at 50 miles per hour and Car Y is traveling at 40 miles per hour.
 - (2) Three minutes ago Car X was $\frac{1}{2}$ mile ahead of Car Y.
103. If a certain animated cartoon consists of a total of 17,280 frames on film, how many minutes will it take to run the cartoon?
- (1) The cartoon runs without interruption at the rate of 24 frames per second.
 - (2) It takes 6 times as long to run the cartoon as it takes to rewind the film, and it takes a total of 14 minutes to do both.

104. At what speed was a train traveling on a trip when it had completed half of the total distance of the trip?
- (1) The trip was 460 miles long and took 4 hours to complete.
 - (2) The train traveled at an average rate of 115 miles per hour on the trip.

105. Tom, Jane, and Sue each purchased a new house. The average (arithmetic mean) price of the three houses was \$120,000. What was the median price of the three houses?
- (1) The price of Tom's house was \$110,000.
 - (2) The price of Jane's house was \$120,000.

106. If x and y are integers, is xy even?
- (1) $x = y + 1$
 - (2) $\frac{x}{y}$ is an even integer.

107. A box contains only red chips, white chips, and blue chips. If a chip is randomly selected from the box, what is the probability that the chip will be either white or blue?
- (1) The probability that the chip will be blue is $\frac{1}{5}$.
 - (2) The probability that the chip will be red is $\frac{1}{3}$.



108. If the successive tick marks shown on the number line above are equally spaced and if x and y are the numbers designating the end points of intervals as shown, what is the value of y ?
- (1) $x = \frac{1}{2}$
 - (2) $y - x = \frac{2}{3}$

109. In triangle ABC , point X is the midpoint of side AC and point Y is the midpoint of side BC . If point R is the midpoint of line segment XC and if point S is the midpoint of line segment YC , what is the area of triangular region RCS ?

- (1) The area of triangular region ABX is 32.
- (2) The length of one of the altitudes of triangle ABC is 8.

110. The product of the units digit, the tens digit, and the hundreds digit of the positive integer m is 96. What is the units digit of m ?

- (1) m is odd.
- (2) The hundreds digit of m is 8.

111. A department manager distributed a number of pens, pencils, and pads among the staff in the department, with each staff member receiving x pens, y pencils, and z pads. How many staff members were in the department?

- (1) The numbers of pens, pencils, and pads that each staff member received were in the ratio 2:3:4, respectively.
- (2) The manager distributed a total of 18 pens, 27 pencils, and 36 pads.

112. Machines X and Y produced identical bottles at different constant rates. Machine X , operating alone for 4 hours, filled part of a production lot; then Machine Y , operating alone for 3 hours, filled the rest of this lot. How many hours would it have taken Machine X operating alone to fill the entire production lot?

- (1) Machine X produced 30 bottles per minute.
- (2) Machine X produced twice as many bottles in 4 hours as Machine Y produced in 3 hours.

113. On a company-sponsored cruise, $\frac{2}{3}$ of the passengers were company employees and the remaining passengers were their guests. If $\frac{3}{4}$ of the company-employee passengers were managers, what was the number of company-employee passengers who were NOT managers?

- (1) There were 690 passengers on the cruise.
- (2) There were 230 passengers who were guests of the company employees.

114. The length of the edging that surrounds circular garden K is $\frac{1}{2}$ the length of the edging that surrounds circular garden G . What is the area of garden K ? (Assume that the edging has negligible width.)

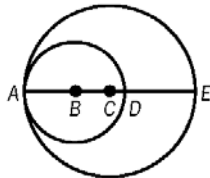
- (1) The area of G is 25π square meters.
- (2) The edging around G is 10π meters long.

115. For any integers x and y , $\min(x, y)$ and $\max(x, y)$ denote the minimum and the maximum of x and y , respectively. For example, $\min(5, 2) = 2$ and $\max(5, 2) = 5$. For the integer w , what is the value of $\min(10, w)$?

- (1) $w = \max(20, z)$ for some integer z .
- (2) $w = \max(10, w)$

116. During a 6-day local trade show, the least number of people registered in a single day was 80. Was the average (arithmetic mean) number of people registered per day for the 6 days greater than 90?

- (1) For the 4 days with the greatest number of people registered, the average (arithmetic mean) number registered per day was 100.
- (2) For the 3 days with the smallest number of people registered, the average (arithmetic mean) number registered per day was 85.



117. In the figure above, points A , B , C , D , and E lie on a line. A is on both circles, B is the center of the smaller circle, C is the center of the larger circle, D is on the smaller circle, and E is on the larger circle. What is the area of the region inside the larger circle and outside the smaller circle?

- (1) $AB = 3$ and $BC = 2$
- (2) $CD = 1$ and $DE = 4$

118. An employee is paid 1.5 times the regular hourly rate for each hour worked in excess of 40 hours per week, excluding Sunday, and 2 times the regular hourly rate for each hour worked on Sunday. How much was the employee paid last week?

- (1) The employee's regular hourly rate is \$10.
- (2) Last week the employee worked a total of 54 hours but did not work more than 8 hours on any day.

119. What was the revenue that a theater received from the sale of 400 tickets, some of which were sold at the full price and the remainder of which were sold at a reduced price?

- (1) The number of tickets sold at the full price was $\frac{1}{4}$ of the total number of tickets sold.
- (2) The full price of a ticket was \$25.

120. The annual rent collected by a corporation from a certain building was x percent more in 1998 than in 1997 and y percent less in 1999 than in 1998. Was the annual rent collected by the corporation from the building more in 1999 than in 1997?

- (1) $x > y$
- (2) $\frac{xy}{100} < x - y$

121. In the xy -plane, region R consists of all the points (x, y) such that $2x + 3y \leq 6$. Is the point (r, s) in region R ?

- (1) $3r + 2s = 6$
- (2) $r < 3$ and $s < 2$

122. What is the volume of a certain rectangular solid?

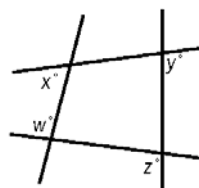
- (1) Two adjacent faces of the solid have areas 15 and 24, respectively.
- (2) Each of two opposite faces of the solid has area 40.

123. Joanna bought only \$0.15 stamps and \$0.29 stamps. How many \$0.15 stamps did she buy?

- (1) She bought \$4.40 worth of stamps.
- (2) She bought an equal number of \$0.15 stamps and \$0.29 stamps.

	Favorable	Unfavorable	Not Sure
Candidate M	40	20	40
Candidate N	30	35	35

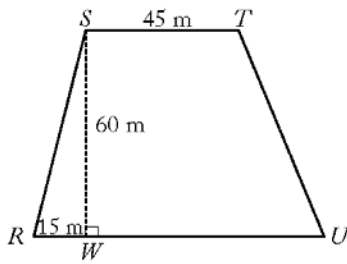
124. The table above shows the results of a survey of 100 voters who each responded "Favorable" or "Unfavorable" or "Not Sure" when asked about their impressions of Candidate M and of Candidate N. What was the number of voters who responded "Favorable" for both candidates?
- The number of voters who did not respond "Favorable" for either candidate was 40.
 - The number of voters who responded "Unfavorable" for both candidates was 10.
125. If \circ represents one of the operations $+$, $-$, and \times , is $k \circ (\ell + m) = (k \circ \ell) + (k \circ m)$ for all numbers k , ℓ , and m ?
- $k \circ 1$ is not equal to $1 \circ k$ for some numbers k .
 - \circ represents subtraction.
126. How many of the 60 cars sold last month by a certain dealer had neither power windows nor a stereo?
- Of the 60 cars sold, 20 had a stereo but not power windows.
 - Of the 60 cars sold, 30 had both power windows and a stereo.
127. In Jefferson School, 300 students study French or Spanish or both. If 100 of these students do not study French, how many of these students study both French and Spanish?
- Of the 300 students, 60 do not study Spanish.
 - A total of 240 of the students study Spanish.
128. A school administrator will assign each student in a group of n students to one of m classrooms. If $3 < m < 13 < n$, is it possible to assign each of the n students to one of the m classrooms so that each classroom has the same number of students assigned to it?
- It is possible to assign each of $3n$ students to one of m classrooms so that each classroom has the same number of students assigned to it.
 - It is possible to assign each of $13n$ students to one of m classrooms so that each classroom has the same number of students assigned to it.
129. What is the median number of employees assigned per project for the projects at Company Z?
- 25 percent of the projects at Company Z have 4 or more employees assigned to each project.
 - 35 percent of the projects at Company Z have 2 or fewer employees assigned to each project.
130. If Juan had a doctor's appointment on a certain day, was the appointment on a Wednesday?
- Exactly 60 hours before the appointment, it was Monday.
 - The appointment was between 1:00 p.m. and 9:00 p.m.
131. When a player in a certain game tossed a coin a number of times, 4 more heads than tails resulted. Heads or tails resulted each time the player tossed the coin. How many times did heads result?
- The player tossed the coin 24 times.
 - The player received 3 points each time heads resulted and 1 point each time tails resulted, for a total of 52 points.



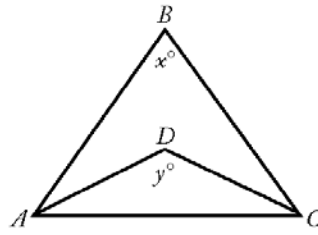
132. What is the value of $x + y$ in the figure above?
- $w = 95$
 - $z = 125$

133. Are all of the numbers in a certain list of 15 numbers equal?
- The sum of all the numbers in the list is 60.
 - The sum of any 3 numbers in the list is 12.

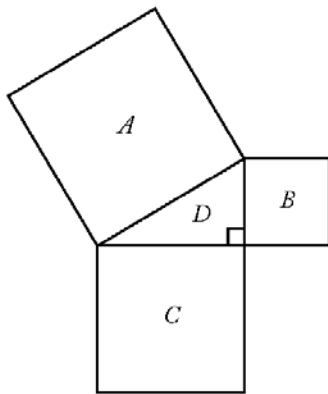
134. A scientist recorded the number of eggs in each of 10 birds' nests. What was the standard deviation of the numbers of eggs in the 10 nests?
- The average (arithmetic mean) number of eggs for the 10 nests was 4.
 - Each of the 10 nests contained the same number of eggs.



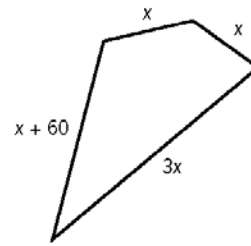
135. Quadrilateral $RSTU$ shown above is a site plan for a parking lot in which side RU is parallel to side ST and RU is longer than ST . What is the area of the parking lot?
- (1) $RU = 80$ meters
 - (2) $TU = 20\sqrt{10}$ meters
136. If the average (arithmetic mean) of six numbers is 75, how many of the numbers are equal to 75?
- (1) None of the six numbers is less than 75.
 - (2) None of the six numbers is greater than 75.
137. At a bakery, all donuts are priced equally and all bagels are priced equally. What is the total price of 5 donuts and 3 bagels at the bakery?
- (1) At the bakery, the total price of 10 donuts and 6 bagels is \$12.90.
 - (2) At the bakery, the price of a donut is \$0.15 less than the price of a bagel.
138. What was the total amount of revenue that a theater received from the sale of 400 tickets, some of which were sold at x percent of full price and the rest of which were sold at full price?
- (1) $x = 50$
 - (2) Full-price tickets sold for \$20 each.
139. Any decimal that has only a finite number of nonzero digits is a terminating decimal. For example, 24, 0.82, and 5.096 are three terminating decimals. If r and s are positive integers and the ratio $\frac{r}{s}$ is expressed as a decimal, is $\frac{r}{s}$ a terminating decimal?
- (1) $90 < r < 100$
 - (2) $s = 4$



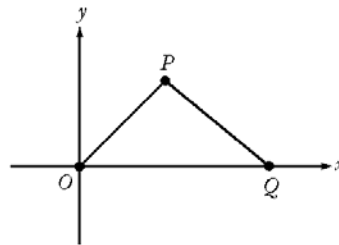
140. In the figure above, what is the value of $x + y$?
- (1) $x = 70$
 - (2) $\triangle ABC$ and $\triangle ADC$ are both isosceles triangles.
141. Committee X and Committee Y, which have no common members, will combine to form Committee Z. Does Committee X have more members than Committee Y?
- (1) The average (arithmetic mean) age of the members of Committee X is 25.7 years and the average age of the members of Committee Y is 29.3 years.
 - (2) The average (arithmetic mean) age of the members of Committee Z will be 26.6 years.
142. What amount did Jean earn from the commission on her sales in the first half of 1988?
- (1) In 1988 Jean's commission was 5 percent of the total amount of her sales.
 - (2) The amount of Jean's sales in the second half of 1988 averaged \$10,000 per month more than in the first half.
143. The price per share of Stock X increased by 10 percent over the same time period that the price per share of Stock Y decreased by 10 percent. The reduced price per share of Stock Y was what percent of the original price per share of Stock X?
- (1) The increased price per share of Stock X was equal to the original price per share of Stock Y.
 - (2) The increase in the price per share of Stock X was $\frac{10}{11}$ the decrease in the price per share of Stock Y.



144. In the figure above, if the area of triangular region D is 4, what is the length of a side of square region A ?
- (1) The area of square region B is 9.
 - (2) The area of square region C is $\frac{64}{9}$.
145. If Sara's age is exactly twice Bill's age, what is Sara's age?
- (1) Four years ago, Sara's age was exactly 3 times Bill's age.
 - (2) Eight years from now, Sara's age will be exactly 1.5 times Bill's age.
146. A report consisting of 2,600 words is divided into 23 paragraphs. A 2-paragraph preface is then added to the report. Is the average (arithmetic mean) number of words per paragraph for all 25 paragraphs less than 120?
- (1) Each paragraph of the preface has more than 100 words.
 - (2) Each paragraph of the preface has fewer than 150 words.
147. A certain bookcase has 2 shelves of books. On the upper shelf, the book with the greatest number of pages has 400 pages. On the lower shelf, the book with the least number of pages has 475 pages. What is the median number of pages for all of the books on the 2 shelves?
- (1) There are 25 books on the upper shelf.
 - (2) There are 24 books on the lower shelf.



148. The figure above shows the number of meters in the lengths of the four sides of a jogging path. What is the total distance around the path?
- (1) One of the sides of the path is 120 meters long.
 - (2) One of the sides of the path is twice as long as each of the two shortest sides.



149. In the rectangular coordinate system above, if $OP < PQ$, is the area of region OPQ greater than 48?
- (1) The coordinates of point P are (6,8).
 - (2) The coordinates of point Q are (13,0).

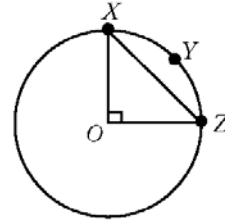
$$S = \frac{\frac{2}{n}}{\frac{1}{x} + \frac{2}{3x}}$$

150. In the expression above, if $xn \neq 0$, what is the value of S ?
- (1) $x = 2n$
 - (2) $n = \frac{1}{2}$
151. If n is a positive integer and $k = 5.1 \times 10^n$, what is the value of k ?
- (1) $6,000 < k < 500,000$
 - (2) $k^2 = 2.601 \times 10^9$

152. If Carmen had 12 more tapes, she would have twice as many tapes as Rafael. Does Carmen have fewer tapes than Rafael?
- (1) Rafael has more than 5 tapes.
 - (2) Carmen has fewer than 12 tapes.
153. If x is an integer, is $x|x| < 2^x$?
- (1) $x < 0$
 - (2) $x = -10$
154. If n is a positive integer, is the value of $b - a$ at least twice the value of $3^n - 2^n$?
- (1) $a = 2^{n+1}$ and $b = 3^{n+1}$
 - (2) $n = 3$
155. The inflation index for the year 1989 relative to the year 1970 was 3.56, indicating that, on the average, for each dollar spent in 1970 for goods, \$3.56 had to be spent for the same goods in 1989. If the price of a Model K mixer increased precisely according to the inflation index, what was the price of the mixer in 1970?
- (1) The price of the Model K mixer was \$102.40 more in 1989 than in 1970.
 - (2) The price of the Model K mixer was \$142.40 in 1989.
156. Is 5^k less than 1,000?
- (1) $5^{k+1} > 3,000$
 - (2) $5^{k-1} = 5^k - 500$
157. The hypotenuse of a right triangle is 10 cm. What is the perimeter, in centimeters, of the triangle?
- (1) The area of the triangle is 25 square centimeters.
 - (2) The 2 legs of the triangle are of equal length.
158. Every member of a certain club volunteers to contribute equally to the purchase of a \$60 gift certificate. How many members does the club have?
- (1) Each member's contribution is to be \$4.
 - (2) If 5 club members fail to contribute, the share of each contributing member will increase by \$2.

159. If $x < 0$, is $y > 0$?

- (1) $\frac{x}{y} < 0$
- (2) $y - x > 0$



160. What is the circumference of the circle above with center O ?
- (1) The perimeter of $\triangle OXZ$ is $20 + 10\sqrt{2}$.
 - (2) The length of arc XYZ is 5π .
161. Beginning in January of last year, Carl made deposits of \$120 into his account on the 15th of each month for several consecutive months and then made withdrawals of \$50 from the account on the 15th of each of the remaining months of last year. There were no other transactions in the account last year. If the closing balance of Carl's account for May of last year was \$2,600, what was the range of the monthly closing balances of Carl's account last year?
- (1) Last year the closing balance of Carl's account for April was less than \$2,625.
 - (2) Last year the closing balance of Carl's account for June was less than \$2,675.
162. If n and k are positive integers, is $\sqrt{n+k} > 2\sqrt{n}$?
- (1) $k > 3n$
 - (2) $n + k > 3n$
163. In a certain business, production index p is directly proportional to efficiency index e , which is in turn directly proportional to investment index i . What is p if $i = 70$?
- (1) $e = 0.5$ whenever $i = 60$.
 - (2) $p = 2.0$ whenever $i = 50$.

164. In the rectangular coordinate system, are the points (r,s) and (u,v) equidistant from the origin?

- (1) $r + s = 1$
 (2) $u = 1 - r$ and $v = 1 - s$

165. If x is an integer, is $9^x + 9^{-x} = b$?

- (1) $3^x + 3^{-x} = \sqrt{b+2}$
 (2) $x > 0$

166. If n is a positive integer, is $\left(\frac{1}{10}\right)^n < 0.01$?

- (1) $n > 2$
 (2) $\left(\frac{1}{10}\right)^{n-1} < 0.1$

167. If n is a positive integer, what is the tens digit of n ?

- (1) The hundreds digit of $10n$ is 6.
 (2) The tens digit of $n + 1$ is 7.

168. What is the value of $\frac{2t+t-x}{t-x}$?

- (1) $\frac{2t}{t-x} = 3$
 (2) $t - x = 5$

169. Is n an integer?

- (1) n^2 is an integer.
 (2) \sqrt{n} is an integer.

170. If n is a positive integer, is $n^3 - n$ divisible by 4?

- (1) $n = 2k + 1$, where k is an integer.
 (2) $n^2 + n$ is divisible by 6.

171. What is the tens digit of positive integer x ?

- (1) x divided by 100 has a remainder of 30.
 (2) x divided by 110 has a remainder of 30.

172. If x , y , and z are positive integers, is $x - y$ odd?

- (1) $x = z^2$
 (2) $y = (z - 1)^2$



173. If arc PQR above is a semicircle, what is the length of diameter \overline{PR} ?

- (1) $a = 4$
 (2) $b = 1$

174. Marcia's bucket can hold a maximum of how many liters of water?

- (1) The bucket currently contains 9 liters of water.
 (2) If 3 liters of water are added to the bucket when it is half full of water, the amount of water in the bucket will increase by $\frac{1}{3}$.

参考答案

1.	B	36.	B	71.	C	106.	D	141.	C
2.	E	37.	A	72.	A	107.	B	142.	E
3.	A	38.	C	73.	B	108.	D	143.	D
4.	E	39.	D	74.	A	109.	A	144.	D
5.	E	40.	B	75.	A	110.	A	145.	D
6.	D	41.	E	76.	E	111.	E	146.	B
7.	A	42.	A	77.	D	112.	B	147.	C
8.	C	43.	E	78.	C	113.	D	148.	B
9.	A	44.	C	79.	E	114.	D	149.	A
10.	B	45.	D	80.	D	115.	D	150.	A
11.	A	46.	B	81.	C	116.	A	151.	D
12.	B	47.	A	82.	B	117.	D	152.	B
13.	D	48.	C	83.	C	118.	E	153.	D
14.	C	49.	B	84.	D	119.	E	154.	A
15.	B	50.	C	85.	A	120.	B	155.	D
16.	A	51.	B	86.	A	121.	E	156.	B
17.	D	52.	B	87.	E	122.	C	157.	D
18.	B	53.	A	88.	B	123.	A	158.	D
19.	C	54.	C	89.	C	124.	A	159.	A
20.	A	55.	E	90.	A	125.	D	160.	D
21.	E	56.	A	91.	E	126.	E	161.	C
22.	E	57.	D	92.	A	127.	D	162.	A
23.	B	58.	E	93.	D	128.	B	163.	B
24.	D	59.	E	94.	A	129.	C	164.	C
25.	D	60.	D	95.	E	130.	C	165.	A
26.	D	61.	C	96.	A	131.	D	166.	D
27.	D	62.	D	97.	A	132.	C	167.	A
28.	E	63.	C	98.	B	133.	B	168.	A
29.	E	64.	B	99.	D	134.	B	169.	B
30.	B	65.	D	100.	D	135.	D	170.	A
31.	B	66.	C	101.	B	136.	D	171.	A
32.	E	67.	D	102.	D	137.	A	172.	C
33.	C	68.	A	103.	D	138.	E	173.	D
34.	D	69.	A	104.	E	139.	B	174.	B
35.	D	70.	D	105.	B	140.	E		

第 10 章 Data Sufficiency 补充题

SECTION 1

- Is x smaller than 2?
 - $7x^2=21$
 - $x^2-6x+5=0$
- If a , b , and c are positive integers, can the product of them be multiple of 24?
 - a , b , c are consecutive integers.
 - a is an even number.
- What is the standard deviation of the three number $x+5$, $y+5$, $z+5$?
 - The standard deviation of x and y is greater than 0.3.
 - The standard deviation of x , y and z equals to 0.2
- What is the greatest common divisor of positive integers m and n ?
 - m is a prime number, $n=2$.
 - m and n are consecutive integers.
- What is the units digit of positive integer x ?
 - x divided by 9 has a remainder of 5.
 - x divided by 3 has a remainder of 2.
- If $z_1, z_2, z_3, \dots, z_n$ is a series of consecutive positive integers, is the sum of all these integers odd?
 - $\frac{z_1+z_2+z_3+\dots+z_n}{n}$ is an odd integer.
 - n is odd.
- In the equation $b=ka+3$, k is a constant. If the possible solutions are in the form (a, b) , is $(2, 3)$ a solution to the equation?
 - $(1, 4)$ is a solution of the equation $b=ka+2$.
 - $(3, 5)$ is a solution of the equation $b=ka+1$.
- Is $ax < a+x$?
 - $0 < x < 1$
 - a is a positive integer.
- If x , y are positive integers, is xy divisible by 4?
 - $y+2$ is divisible by 4.
 - $x+2$ is divisible by 4.
- The average (arithmetic mean) of M consecutive odd integers is 10. What is the smallest number in the sequence?
 - The range of the M consecutive odd integers is 14.
 - The greatest number of the M consecutive odd integers is 15.
- What is the greatest probability that events R and W will both occur?
 - The probability that events R will occur is 0.38.
 - The probability that events W will occur is 0.46.
- Sandra saves the same amount of money each month in an account for her children future college expenses. Ten years from now, will Sandra have saved at least \$12,000 more than she currently has in the account?
 - The years from now, Sandra will have saved a total of \$20,000 in the account.
 - Sandra will have saved \$ 110 in the account this month.
- Alan and Betty live in a multi-story

apartment building. How many stories does the building have?

- (1) There are 3 stories between those on which Alan and Betty live.
- (2) There are 9 stories above Alan's floor and 9 stories below Betty's floor

SECTION 2

1. What is the value of y ?
 - (1) y is an odd integer between 28 and 34
 - (2) $31 < y < 36$
2. The price of a television set was reduced by 25 percent. What was its original price?
 - (1) The reduced price was \$187.50.
 - (2) The original price exceeded the reduced price by more than \$60.00.
3. What is the remainder when the positive integer n is divided by 6?
 - (1) n is a multiple of 5.
 - (2) n is a multiple of 12.
4. Three friends rented a car for a week and divided the cost equally. What was the total cost of renting the car?
 - (1) If the three friends had kept the car for a second week, they could have obtained the two-week rate, which was 1.5 times the cost of a one-week rental.
 - (2) If a fourth friend had joined the three friends and the cost had been divided equally among the four friends, the cost to each of the original three would

have been reduced by \$15?

5. Is $x > y$?
 - (1) $x = y + 2$
 - (2) $\frac{x}{2} = y - 1$
6. Sally gave some of her candy to her friends. How many pieces of candy did she have before giving any to her friends?
 - (1) Sally gave each friend 8 pieces of candy.
 - (2) Sally had 7 pieces of candy left after giving candy to her friends.
7. What was Jean's insurance premium in 1995?
 - (1) The ratio of Jean's insurance premium in 1995 to her insurance premium in 1994 was $\frac{6}{5}$.
 - (2) Jean's insurance premium in 1995 was 20 percent more than her insurance premium in 1994.
8. What is the average (arithmetic mean) of x and y ?
 - (1) The average of x and $2y$ is 10.
 - (2) The average of $2x$ and $7y$ is 32.

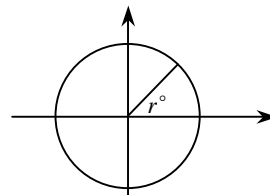


图 9.1

9. The figure above shows the present position on a radar screen of a sweeping beam that is rotating at a constant rate in

- a clockwise direction. In which of the four quadrants will the beam lie 30 seconds from now?
- (1) In each 30-second period, the beam sweeps through 3.690°
- (2) $r = 40$
10. The number of seats in the first row of an auditorium is 18 and the number of seats in each row thereafter is 2 more than in the previous row. What is the total number of seats in the auditorium?
- (1) The number of rows of seats in the auditorium is 27.
- (2) The number of seats in the last row is 70.
11. How many books did a librarian purchase?
- (1) The librarian paid an average (arithmetic mean) of \$15 per book for the books purchased.
- (2) The total sales tax on the books purchased was \$7.
12. Is the integer n a multiple of 15?
- (1) n is a multiple of 20.
- (2) $n + 6$ is a multiple of 3.
13. What is the area of rectangular region R ?
- (1) Each diagonal of R has length 5.
- (2) The perimeter of R is 14.
14. In a certain coding scheme, each word is encoded by replacing each letter in the word with another letter. The same code is used for all words, so that the same letter replaces a given letter each time the given letter occurs. What code will result when the word TAME is encoded by this scheme?
- (1) When the word MAT is encoded, the result is DLX.
- (2) When the word TEA is encoded, the result is XRL.
15. A certain high school with a total enrollment of 900 students held a science fair for three days last week. How many of the students enrolled in the high school attended the science fair on all three days?
- (1) Of the students enrolled in the school, 30 percent attended the science fair on two or more days.
- (2) Of the students enrolled in the school, 10 percent of those that attended the science fair on at least one day attended on all three days.
16. What is the probability that events A and B both occur?
- (1) The probability that event A occurs is 0.8.
- (2) The probability that event B occurs is 0.6.
17. If $n = \frac{p}{q}$ where p and q are nonzero integers, is n an integer?
- (1) n^2 is an integer.
- (2) $\frac{2n+4}{2}$ is an integer.
18. In the rectangular coordinate system, are the points (r, s) and (u, v) equidistant from the origin?
- (1) $r + s = 1$
- (2) $u = 1 - r$ and $v = 1 - s$

19. Is $ab = 1$?

(1) $aba = a$

(2) $bab = b$

20. Is $\sqrt{(x-3)^2} = 3-x$?

(1) $x \neq 3$

(2) $-x/x > 0$

SECTION 3

1. How much is 20 percent of a certain number?

(1) 10 percent of the number is 5.

(2) 40 percent of twice the number is 40.

2. Is r greater than 0.27?

(1) r is greater than $\frac{1}{4}$

(2) r is equal to $\frac{3}{10}$

3. What percent of a group of people are women with red hair?

(1) Of the women in the group, 5 percent have red hair.

(2) Of the men in the group, 10 percent have red hair.

4. If i and j are integers, is $i + j$ an even integer?

(1) $i < 10$

(2) $i = j$

TOTAL EXPENSES FOR THE FIVE DIVISIONS OF COMPANY

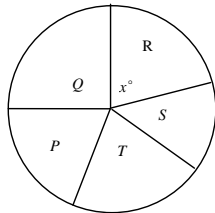


图 9.2

5. The figure above represents a circle graph of Company H's total expenses broken down by the expenses for each of its five divisions. If O is the center of the circle and if Company H's total expenses are \$5,400,000, what are the expenses for division R?

(1) $x = 94$

(2) The total expenses for divisions S and T are twice as much as the expenses for division R.

6. What is the value of $\frac{1}{k} + \frac{1}{r}$?

(1) $k + r = 20$

(2) $kr = 64$

7. A number of people each wrote down one of the first 30 positive integers. Were any of the integers written down by more than one of the people?

(1) The number of people who wrote down an integer was greater than 40.

(2) The number of people who wrote down an integer was less than 70.

8. If d denotes a decimal, is $d \geq 0.5$?

(1) When d is rounded to the nearest tenth, the result is 0.5.

(2) When d is rounded to the nearest integer, the result is 1.

9. How many of the 60 cars sold last month by a certain dealer had neither power windows nor a stereo?

(1) Of the 60 cars sold, 20 had a stereo

but not power windows.

- (2) Of the 60 cars sold, 30 had both power windows and a stereo.
10. In the xy -plane, does the point $(4, 12)$ lie on line k ?
- (1) The point $(1, 7)$ lies on line k .
 (2) The point $(-2, 2)$ lies on line k .
11. What was the total amount of revenue that a theater received from the sale of 400 tickets, some of which were sold at x percent of full price and the rest of which were sold at full price?
- (1) $x = 50$
 (2) Full-price tickets sold for \$20 each.
12. If x , y , and z are nonzero numbers, is $xz = 12$?
- (1) $x^2yz = 12xy$ (2) $\frac{z}{4} = \frac{3}{x}$
13. When a player in a certain game tossed a coin a number of times, 4 more heads than tails resulted. Heads or tails resulted each time the player tossed the coin. How many times did heads result?
- (1) The player tossed the coin 24 times.
 (2) The player received 3 points each time heads resulted and 1 point each time tails resulted, for a total of 52 points.
14. What is the area of triangular region ABC above?
- (1) The product of BD and AC is 20.
 (2) $x = 45$
15. What is the value of $36,500(1.05)^n$?
- (1) $n^2 - 5n + 6 = 0$
 (2) $n - 2 \neq 0$
16. The inflation index for the year 1989 relative to the year 1970 was 3.56, indicating that, on the average, for each dollar spent in 1970 for goods, \$3.56 had to be spent for the same goods in 1989. If the price of a Model K mixer increased precisely according to the inflation index, what was the price of the mixer in 1970?
- (1) The price of the Model K mixer was \$102.40 more in 1989 than in 1970.
 (2) The price of the Model K mixer was \$142.40 in 1989.
17. Every member of a certain club volunteers to contribute equally to the purchase of a \$60 gift certificate. How many members does the club have?
- (1) Each member's contribution is to be \$4.
 (2) If 5 club members fail to contribute, the share of each contributing member will increase by \$2.
18. While driving on the expressway, did Robin ever exceed the 55-miles-per-hour speed limit?
- (1) Robin drove 100 miles on the expressway.
 (2) Robin drove for 2 hours on the expressway.

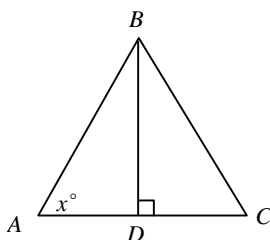


图 9.3

19. Is n an integer?

- (1) n^2 is an integer.
 (2) \sqrt{n} is an integer.

20. Is x negative?

- (1) $n^3(1-x^2) < 0$
 (2) $x^2 - 1 < 0$

SECTION 4

1. What is 5 percent of x ?

- (1) $x = 200$
 (2) 10 percent of x is 20.

2. For which type of investment, J or K, is the annual rate of return greater?

- (1) Type J returns \$115 per \$1,000 invested for any one-year period and type K returns \$300 per \$2,500 invested for any one-year period.
 (2) The annual rate of return for an investment of type K is 12 percent.

3. What is the value of $\left(\frac{a}{b}\right)^n$

- (1) $a^n = 32$ (2) $b^n = 243$

4. What is the tenths digit of the decimal d ?

- (1) $d = \frac{7}{25}$
 (2) d is 28 percent of 1.

5. Is $a = 0$?

- (1) $ab = 0$ (2) $a^2 = 0$

6. What percent of the drama club members enrolled at a certain school are female students?

- (1) Of the female students enrolled at the

school, 40 percent are members of the drama club.

- (2) Of the male students enrolled at the school, 25 percent are members of the drama club.

7. What is the perimeter of rectangle R?

- (1) R is a square.
 (2) The area of R is 36.

8. Carmen currently works 30 hours per week at her part-time job. If her gross hourly wage were to increase by \$1.50, how many fewer hours could she work per week and still earn the same gross weekly pay as before the increase?

- (1) Her gross weekly pay is currently \$225.00.
 (2) An increase of \$1.50 would represent an increase of 20 percent of her current gross hourly wage.

9. In a serving of a breakfast cereal that contains only oats, raisins, and nuts, the ratio of oats to raisins to nuts by weight is 3 to 2 to 1, respectively.

How many ounces of raisins are in the serving?

- (1) The serving weighs 4 ounces.
 (2) The serving contains 2 ounces of oats.

10. Is $x > 0$

- (1) $x^2 > 0$ (2) $x^3 > 0$

11. A taxi company charges f cents for the first mile of a taxi ride and m cents for each additional mile. How much does

- the company charge for a 10-mile taxi ride?
- (1) The company charges \$0.90 for a 2-mile ride.
- (2) The company charges \$1.20 for a 4-mile ride.
12. What was the discounted price of a certain television set?
- (1) The original price of the television set was \$50 more than the discounted price.
- (2) The original price of the television set was 110 percent of the discounted price.
13. If $x > 0$, is $\frac{x}{y} > x$?
- (1) $0 < y < 1$
- (2) $x > 1$
14. Is n an integer?
- (1) $2n$ is an integer.
- (2) $\frac{n}{2}$ is an integer.
15. At what speed was a train traveling on a trip when it had completed half of the total distance of the trip?
- (1) The trip was 460 miles long and took 4 hours to complete.
- (2) The train traveled at an average rate of 115 miles per hour on the trip.
16. If r and s are integers, is $r^2 + s$ even?
- (1) The product rs is odd.
- (2) r is odd.
17. What is the volume of a certain rectangular solid?
- (1) Two adjacent faces of the solid have areas 15 and 24, respectively.
- (2) Each of two opposite faces of the solid has area 40.
18. Is $x > y$?
- (1) $ax > ay$
- (2) $a^2x > a^2y$
19. If n is a positive integer and $k = 5.1 \times 10^n$, what is the value of k ?
- (1) $6,000 < k < 500,000$
- (2) $k^2 = 2.601 \times 10^9$
20. What is the average (arithmetic mean) of $3x$ and $6y$?
- (1) $x + 2y = 7$
- (2) $x + y = 5$

SECTION 5

1. What is the capacity, in liters, of a certain aquarium?
- (1) Three liters is $\frac{1}{2}$ of the capacity of the aquarium.
- (2) One-half liter is $\frac{1}{12}$ of the capacity of the aquarium.
2. What is the value of n ?
- (1) $\sqrt{n} = 2$
- (2) $\sqrt{4n} = 4$
3. In $\triangle PQR$, what is the measure of angle P ?
- (1) Angle Q is a right angle.
- (2) The measure of angle R is 17°
4. What amount did Jean earn from the commission on her sales in the first half of 1988?

- (1) In 1988 Jean's commission was 5 percent of the total amount of her sales.
- (2) The amount of Jean's sales in the second half of 1988 averaged \$10,000 per month more than in the first half.
5. A certain car traveled from one town to another without stopping. What was the car's average speed for the trip?
- (1) The car traveled the 90-mile trip in 2 hours.
- (2) The car traveled the first 40 miles of the trip in 1 hour.
6. What is the value of x ?
- (1) $2x + 3y = 12$
- (2) $5x + 7y = 29$
7. Does $x = y$?
- (1) $|x| = |y|$
- (2) $x^2 = y^2$
8. Dan took a 20-question multiple-choice test in psychology. If Dan answered every question, did he answer at least 12 questions correctly?
- (1) Dan answered fewer than 40 percent of the questions incorrectly.
- (2) Dan answered at least 25 percent of the questions incorrectly.

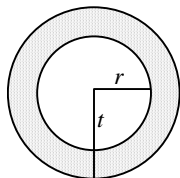


图 9.4

9. The figure above shows the circular cross section of a concrete water pipe. If the inside radius of the pipe is r feet and the outside radius of the pipe is t feet, what is the value of r ?
- (1) The ratio of $t - r$ to r is 0.15 and $t - r$ is equal to 0.3 foot.
- (2) The area of the concrete in the cross section is 1.29π square feet.
10. If $a < x < b$ and $c < y < d$, is $x < y$?
- (1) $a < c$
- (2) $b < c$
11. Is $\frac{3x+8}{x+2}$ an integer?
- (1) x is an integer.
- (2) $x = 0$
12. How many people did Apex Company employ in 1990?
- (1) The company employed 538 more people in 1991 than in 1990.
- (2) The company employed 20 percent more people in 1991 than in 1990.

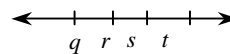


图 9.5

13. Of the four numbers represented on the number line above, is r closest to zero?
- (1) $q = -s$
- (2) $-t < q$
14. Is the integer n divisible by 20?
- (1) n is divisible by 5.
- (2) n is divisible by 6.
15. If Mark saved an average (arithmetic mean)

of \$80 per week for 3 consecutive weeks,
how much did he save the second week?

- (1) The average amount that Mark saved per week for the first 2 weeks was \$60.
(2) The amount that Mark saved the first week was $\frac{1}{2}$ the amount he saved the second week and $\frac{1}{3}$ the amount he saved the third week.

16. If p and q are positive integers, what is the value of q ?

- (1) $q^{p-1} = 1$
(2) $p = 1$

17. If $x \neq -1$, which is greater, $\frac{1}{x+1}$ or $\frac{x}{2}$?

- (1) $x \geq 0$
(2) $x < 3$

18. In a certain two-digit integer, the ratio of the units digit to the tens digit is 2 to 3.

What is the integer?

- (1) The tens digit is 3 more than the units digit.
(2) The product of the two digits is 54.

19. If Carmen had 12 more tapes, she would have twice as many tapes as Rafael.
Does Carmen have fewer tapes than Rafael?

- (1) Rafael has more than 5 tapes.
(2) Carmen has fewer than 12 tapes.

20. What is the value of $\frac{2t+t-x}{t-x}$?

- (1) $\frac{2t}{t-x} = 3$
(2) $t-x = 5$

参考答案:

SECTION 1

- 1~5 ACBBE
6~10 ADCCD
11~13 CBE

SECTION 2

- 1~5 CABBA
6~10 EECCD
11~15 ECCCE
16~20 EDCEB

SECTION 3

- 1~5 DBEBA
6~10 CABEC
11~15 EDDAC
16~20 DDEBE

SECTION 4

- 1~5 DACDB
6~10 ECDDB
11~15 CCABE
16~20 ACBDA

SECTION 5

- 1~5 DDCEA
6~10 CEAAB
11~15 BCAEB
16~20 EEDBA

第 11 章 三套模拟题

SECTION 1

- What is the total number of different 5-digit integers that contain all of the digits 2, 5, 4, 3, 7 and which both the units digit and thousands digit are odd integers?
(A) 10
(B) 8
(C) 36
(D) 24
(E) 12
- How much longer is the edge of a cube with a surface area of 1350 than the edge of a cube with a surface area of 600?
(A) 5
(B) 15
(C) 150
(D) 250
(E) 750
- Bernie has already read $\frac{1}{5}$ of his biology homework on Monday night. What fraction of his remaining homework would Bernie have to read on Tuesday night in order to have read $\frac{2}{3}$ his original biology assignment?
(A) $\frac{7}{10}$
(B) $\frac{7}{12}$
(C) $\frac{5}{12}$
(D) $\frac{2}{3}$
(E) $\frac{1}{15}$
- The median of five different positive integers is 12, which of the following is the least possible sum of these five positive integers?
(A) 38
(B) 21
(C) 42
(D) 54
(E) 60
- What's the sum of all the digits of integer $(10^{100}-38)$?
(A) 881
(B) 888
(C) 890
(D) 872
(E) 449
- Six different numbers are to be selected from the (0, 1, 2, 3, 4, 5, 6) to compose a 6-digit even number that is greater than 300,000. How many such integers are there?
(A) 720
(B) 1920

- (C) 960
(D) 1680
(E) P_7^6
7. There are 5 cars, on yellow, one blue and three red. The cars are lined up in a row and cars of the same color have no difference. How many different arrangements are there?
(A) 20
(B) 120
(C) 6
(D) 114
(E) 60
8. If points X and Y are two distinct points in the coordinate plane, which of the following can have more than one possible value?
(A) The circumference of a circle with diameter XY .
(B) The area of a square with diagonal XY .
(C) The perimeter of a right isosceles triangle with a leg XY .
(D) The area of a circle with chord XY .
(E) The area of an equilateral triangle with base XY .
9. Both 5^2 and 3^3 are factors of $n \times 2^5 \times 6^2 \times 7^3$ where n is a positive integer. What is the smallest possible value of n ?
(A) 25
(B) 27
(C) 45
(D) 75
(E) 125
10. Two cubes each have 6 sides, which represents 1,2,3,4,5,6 respectively. Put the two cubes on the desk, how many possibilities are there that the sum of the numbers on the two upsides is even?
(A) 9
(B) 6
(C) 18
(D) 24
(E) 12
11. What is the greatest possible common divisor of two different positive integers, which are less than 256?
(A) 255
(B) 254
(C) 128
(D) 127
(E) 51
12. A marching band of 240 musicians are to march in a rectangular formation with m rows of exactly n musicians each. There can be no less than 8 musicians per row and no more than 30 musicians per row. How many different rectangular formations are possible?
(A) 7
(B) 4
(C) 6
(D) 7
(E) 8
13. If a and b are odd integers, which of the following is an even integer?
(A) $a(b-4)$
(B) $ab+4$
(C) $(a+2)(b-4)$
(D) $3a+5b$
(E) $a(a+6)$

14. $(-\frac{4}{5})^{200}$ is approximate to?
- (A) -1
(B) $\frac{1}{2}$
(C) 1
(D) 0
(E) $-\frac{1}{2}$
15. The probability that event A occurs is 0.6 and the probability that event B occurs is 0.5. What is the greatest value of the probability that events A and B both not occur?
- (A) 0.2
(B) 0.5
(C) 0.4
(D) 0.3
(E) 0.5
16. If x is 1,2,3 or 4 and 7, y is 2, 4 or 8, and then the product xy can have how many different possible values?
- (A) twelve
(B) eleven
(C) nine
(D) seven
(E) eight
17. In a certain flower shop, which stocks four types of flowers, there are $\frac{1}{3}$ as many violets as carnations, and $\frac{1}{2}$ as many tulips as violets. If there are equal numbers of roses and tulips, what percent of the flowers in the shop are carnations?
- (A) 10%
(B) 33%
- (C) 40%
(D) 50%
(E) 60%
18. Is x greater than 75 percent of y ?
- (1) $x = 40$
(2) $y = 50$
19. The integer x is how much greater than 3?
- (1) $10^x = 100,000$
(2) $\frac{1}{10^x} = 0.00001$
20. A citrus fruit grower receives \$15 for each crate of oranges shipped and \$18 for each crate of grapefruit shipped. How many crates of oranges did the grower ship last week?
- (1) Last week the number of crates of oranges that the grower shipped was 20 more than twice the number of crates of grapefruit shipped.
(2) Last week the grower received a total of \$38,700 from the crates of oranges and grapefruit shipped.
21. In the figure below, what is the length of AC ?
- (1) $x + y = 13$
(2) $xy = 36$

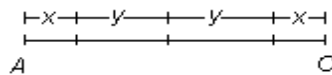


图 10.1

22. The charge for a telephone call between City R and City S is \$0.42 for each of the first 3 minutes and \$0.18 for each additional minute. A certain call between

these two cities lasted for x minutes, where x is an integer. How many minutes long was the call?

- (1) The charge for the first 3 minutes of the call was \$0.36 less than the charge for the remainder of the call.
- (2) The total charge for the call was \$2.88.

23. Is the integer P odd?

- (1) The sum of P , $P + 4$, and $P + 11$ is even.
- (2) The sum of $P - 3$, P , and $P + 11$ is odd.

24. If a certain grove consists of 36 pecan trees, what was the yield per tree last year?

- (1) The yield per tree for the 18 trees in the northern half of the grove was 60 kilograms last year.
- (2) The yield per tree for the 18 trees in the eastern half of the grove was 55 kilograms last year.

25. What was the percent increase in the value of a certain antique from January 1, 1981, to December 31, 1981?

- (1) The value of the antique on January 1, 1981, was \$3,000.
- (2) The value of the antique on December 31, 1981, was double the value of the antique on January 1, 1981.

26. In the xy -plane, is point $(2, -3)$ on line λ ?

- (1) Point $(-2, 3)$ is on λ .

(2) λ is not perpendicular to the x -axis.

27. If r is represented by the decimal $0.t5$, what is the digit t ?

- (1) $r < \frac{1}{3}$
- (2) $r < \frac{1}{10}$

28. Is $7 < \sqrt{n} < 8$?

- (1) $n > 50$
- (2) $n < 60$

29. If a total of 84 students are enrolled in two sections of a calculus course, how many of the 84 students are female?

- (1) $\frac{2}{3}$ of the students in Section 1 are female.
- (2) $\frac{1}{2}$ of the students in Section 2 are male.

30. What is the value of the greater of two numbers if one of the numbers is twice the other number?

- (1) One number is 5.
- (2) The sum of the two numbers is 15.

31. If $r > 0$ and $s > 0$, is $\frac{r}{s} < \frac{s}{r}$?

- (1) $\frac{r}{3s} = \frac{1}{4}$
- (2) $s = r + 4$

32. Company R's annual profit has increased by a constant amount each calendar year since 1985. What was Company R's annual profit in 1991?

- (1) In 1985 Company R's annual profit was \$212,000; in 1989 Company R's annual profit was \$242,000.

(2) Company R's annual profit has increased by \$7,500 each year since 1985.

33. If x is an integer, is $\frac{54+27}{x}$ an integer?

(1) $6 \leq x \leq 81$

(2) x is a multiple of 3.

34. The figure below shows the shape of a flowerbed. If arc QR is a semicircle and $PQRS$ is a rectangle with $QR > RS$, what is the perimeter of the flowerbed?

(1) The perimeter of rectangle $PQRS$ is 28 feet.

(2) Each diagonal of rectangle $PQRS$ is 10 feet long.

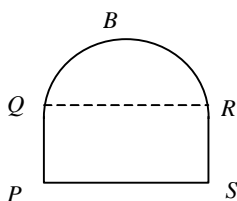


图 10.2

35. If $4x = 5$, $y = 10z$, what is the value of $x + y + z$?

(1) $x - y = 6$

(2) $y + z = 36$

36. Committee X and Committee Y, which have no common members, will combine to form Committee Z. Does Committee X have more members than Committee Y?

(1) The average (arithmetic mean) age of the members of Committee X is 25.7 years and the average age of the members of Committee Y is 29.3 years.

(2) The average (arithmetic mean) age of the members of Committee Z will be 26.6 years.

37. What is the value of y ?

(1) $y^2 - 7y + 12 = 0$

(2) $y > 0$

SECTION 2

1. The numbers in the list above are ordered from least to greatest. If the median is one greater than the arithmetic mean, what is the value of y ?

(A) 11

(B) 12

(C) 10

(D) 13

(E) 15

2. In a certain two-digit number, the tens' digit is triple the units' digit; this two-digit number can have how many different possible values?

(A) 5

(B) 4

(C) 6

(D) 3

(E) 8

3. How many of the positive divisors of 960 are also multiples of 6?

(A) 7

(B) 6

(C) 8

(D) 10

(E) 12

4. The perimeter of a rectangle is 40. What is the greatest area of the rectangle?
- (A) 120
(B) 80
(C) 60
(D) 100
(E) 40
5. How many positive 4-digit integers begin (on the left) with an even digit and end with an odd digit (on the right)?
- (A) 250
(B) 500
(C) 2,000
(D) 2,500
(E) 1,120
6. A certain jar contains 100 jelly beans: 50 white, 30 green, 10 yellow, 5 red, 4 purple, and 1 black. If a jellybean is to be chosen at random, what is the probability that the jellybean will be neither purple nor red?
- (A) 0.09
(B) 0.11
(C) 0.55
(D) 0.91
(E) 0.96
7. How many 3-digit numbers between 100 and 200 have a digit that is the average of the other 2 digits?
- (A) 10
(B) 7
(C) 9
(D) 11
(E) 19
8. The traffic light changes color in the order of Green–Yellow–Red, and the time duration for each color is 45s, 5s, 40s respectively. How many seconds at most can the red light appear in a period of 5 minutes?
- (A) 150s
(B) 120s
(C) 180s
(D) 160s
(E) 100s
9. If each of the following compound fractions is written as a decimal, which will have the largest digit in the hundredth's place?
- (A) $2\frac{1}{3}$
(B) $3\frac{3}{4}$
(C) $4\frac{1}{11}$
(D) $5\frac{4}{13}$
(E) $6\frac{4}{25}$
10. What is the units' digit of $(9)^5 (13)^3 (7)^3$?
- (A) 1
(B) 3
(C) 6
(D) 7
(E) 9
11. In a group of 80 students, 24 are taking physics, 40 are taking mathematics, and 20 are taking both physics and mathematics. If a student were randomly selected from all of the 80 students, what is the probability that the student selected are taking neither physics nor mathematics?

- (A) 0.20
(B) 0.25
(C) 0.45
(D) 0.55
(E) 0.60
12. Select 4 people from 3 boys and 3 girls. What is the probability of two boys and two girls being selected?
(A) $\frac{3}{15}$
(B) $\frac{4}{9}$
(C) $\frac{3}{5}$
(D) $\frac{2}{3}$
(E) $\frac{1}{3}$
13. If the two-digit integers M and N are positive and have the same digits, but in re-verse order. Which of the following CANNOT be the difference of M and N ?
(A) 19197
(B) -99
(C) 43218
(D) 54
(E) 2816
14. If each term of the sequence (a_1, a_2, \dots, a_n) is either 7 or 77, and the sum of the sequence is 350. Which of following could be the value of n ?
(A) 45
(B) 42
(C) 40
(D) 39
(E) 36
15. In a certain game, each player scores either 2 points or 5 points. If n players score 2 points and m players score 5 points, and the total number of points scored is 50, what is the least possible positive difference between n and m ?
(A) 1
(B) 3
(C) 5
(D) 7
(E) 9
16. A circular wheel travels along a flat surface at a constant rate of 2.5 feet per second. How many complete revolutions will the wheel make in 50 seconds if its diameter is 3 feet?
(A) 6
(B) 10
(C) 13
(D) 14
(E) 19
17. Each of 5 positive whole numbers is at least 8. The average (arithmetic mean) of these 5 numbers is 24. If the average of two of the numbers is 18, what is the greatest possible value that any of these 5 numbers can have?
(A) 28
(B) 50
(C) 68
(D) 76
(E) 84
18. The area of a rectangular floor is 56 square feet. If both the length and the width of the rectangular floor are integers, the length can have how many possible values?
(A) 4
(B) 6
(C) 10
(D) 8

- (E) 7
19. In a class of 80 people, Mary's percentile is 90%. In another class of 100 people, 19 people's percentile is greater than Mary's. If the two classes are to be merged into one, what will Mary's percentile be in the new class?
- (A) 75%
(B) 72%
(C) 85%
(D) 72.9%
(E) 81%
20. The result obtained when x is multiplied by y is equal to ten times the result obtained when y is subtracted from x . If y equals 5, what does x equal?
- (A) 50
(B) 25
(C) 15
(D) 10
(E) 5
21. Six people (two of them are John and Marry) are lined in a row. How many arrangements are there if John and Marry are at both ends of the row?
- (A) 720
(B) 30
(C) 48
(D) 2
(E) 24
22. The numbers 3, 10, 17, 24, 31, and 38 are the first six terms of an infinite sequence in which each term after the first is 7 greater than the preceding term. What is the 45th term of the sequence?
- (A) 308
(B) 311
(C) 312
(D) 315
(E) 318
23. A sequence is symmetrically distributed around its arithmetic mean m , and the numbers that fall in the standard deviation d count for 68% of the total numbers in the sequence. What's the percentage of numbers that fall below $m+d$?
- (A) 16%
(B) 68%
(C) 50%
(D) 32%
(E) 84%
24. If the list price of a new car was \$12,300, what was the cost of the car to the dealer?
- (1) The cost to the dealer was equal to 80 percent of the list price.
(2) The car was sold for \$11,070, which was 12.5 percent more than the cost to the dealer.
25. If p , q , x , y , and z are different positive integers, which of the five integers is the median?
- (1) $p + x < q$
(2) $y < z$
26. A certain employee is paid \$6 per hour for an 8-hour workday. If the employee is paid $1\frac{1}{2}$ times this rate for time worked

in excess of 8 hours during a single day, how many hours did the employee work today?

- (1) The employee was paid \$18 more for hours worked today than for hours worked yesterday.
- (2) Yesterday the employee worked 8 hours.

27. If n is a member of the set (33, 36, 38, 39, 41, 42), what is the value of n ?

- (1) n is even.
- (2) n is a multiple of 3.

28. In the figure below, what is the length of AD ?

- (1) $AC = 6$
- (2) $BD = 6$



图 10.3

29. A retailer purchased a television set for x percent less than its list price, and then sold it for y percent less than its list price. What was the list price of the television set?

- (1) $x = 15$
- (2) $x - y = 6$

30. Is x^2 greater than x ?

- (1) x^2 is greater than 1.
- (2) x is greater than -1 .

31. What is the value of $\frac{r}{2} + \frac{s}{2}$?

- (1) $\frac{r+s}{2} = 5$
- (2) $r + s = 10$

32. The circular base of an above-ground swimming pool lies in a level yard and just touches two straight sides of a fence at points A and B, as shown in the figure below. Point C is on the ground where the two sides of the fence meet.

How far from the center of the pool's base is point A?

- (1) The base has area 250 square feet.
- (2) The center of the base is 20 feet from point C

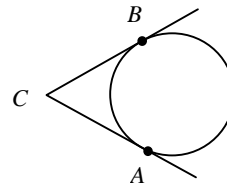


图 10.4

33. In 1979 Mr. Jackson bought a total of n shares of stock X and Mrs. Jackson bought a total of 300 shares of stock X. If the couple held all of their respective shares throughout 1980, and Mr. Jackson's 1980 dividends on his n shares totaled \$150, what was the total amount of Mrs. Jackson's 1980 dividends on her 300 shares?

- (1) In 1980 the annual dividend on each share of stock X was \$0.75.
- (2) In 1979 Mr. Jackson bought a total of 200 shares of stock X.

34. If Sara's age is exactly twice Bill's age, what is Sara's age?

- (1) Four years ago, Sara's age was exactly 3 times Bill's age.
- (2) Eight years from now, Sara's age will be exactly 1.5 times Bill's age.

35. What is the value of $\frac{x}{yz}$?
- (1) $x = \frac{y}{2}$ and $z = \frac{2x}{5}$
- (2) $\frac{x}{z} = \frac{5}{2}$ and $\frac{1}{y} = \frac{1}{10}$
36. An infinite sequence of positive integers is called an "alpha sequence" if the number of even integers in the sequence is finite. If S is an infinite sequence of positive integers, is S an alpha sequence?
- (1) The first ten integers in S are even.
- (2) An infinite number of integers in S are odd.
37. if $xy > 0$, does $(x - 1)(y - 1) = 1$?
- (1) $x + y = xy$
- (2) $x = y$
- all three kind languages. What is the number of the student that can speak none of the language?
- (A) 7
- (B) 10
- (C) 15
- (D) 45
- (E) 25
3. In how many of the whole numbers between 100 and 200 does the digit 6 occur exactly once?
- (A) 18
- (B) 20
- (C) 10
- (D) 9
- (E) 19
4. There are 8 balls in a box. 2 of them are red, 4 of them are white, 2 two of them are blue. If 2 balls are to be simultaneously and randomly selected, what is the probability that they will both be red?
- (A) $\frac{1}{8}$
- (B) $\frac{1}{28}$
- (C) $\frac{1}{4}$
- (D) $\frac{3}{14}$
- (E) $\frac{1}{2}$
5. Two positive integers a and b have only one common divisor, if $\frac{b}{a} + 2 = \frac{18}{7}$, then which of the following could be the value of $(a - b)$?
- (A) 5

SECTION 3

1. There are five cards marked 1, 2, 3, 4, 5 in a box. If the sum of two cards which are taken from those cards randomly (no replacement) is even. What is the probability that these two cards are both odd?
- (A) $\frac{1}{4}$
- (B) $\frac{3}{4}$
- (C) $\frac{3}{10}$
- (D) $\frac{3}{5}$
- (E) $\frac{1}{3}$
2. A language class has 155 students. 80 of them can speak English. 40 of them can speak Russian, 60 of them can speak French. There are 50 of them can speak exactly two languages, and 10 can speak

- (B) 3
(C) 4
(D) 7
(E) 6
6. $P(x, y)$ is the midpoint of AB , $A(1, 3)$, $B(-1, -1)$, then what are the coordinates of P ?
(A) (1,2)
(B) (1.5, 1.5)
(C) (0,1)
(D) (1,0)
(E) (0, -1)
7. In the coordinate plane, points B and C are endpoints of a diameter of circle with center O , if the coordinates of B are (3, 4) and the coordinates of C are (-7, 8), then what are the coordinates of the center?
(A) (2, -5)
(B) (-5, 2)
(C) (0, 0)
(D) (1, 0)
(E) (-2, 6)
8. When the even integer n is divided by 7, the remainder is 5, then what is the remainder when n is divided by 14?
(A) 5
(B) 10
(C) 6
(D) 12
(E) it can not be calculated according to the information above
9. If n is an positive integer, then the units digit of n^{102} cannot be which of the following?
(A) 4
(B) 3
(C) 9
(D) 2
(E) 0
10. Five red balls and two blue balls are placed in an empty box. One ball a time is to be selected randomly and removed 4 balls from the box. What is the probability that the first two balls removed will be red and the third ball removed will be blue?
(A) $\frac{4}{21}$
(B) $\frac{4}{7}$
(C) $\frac{5}{8}$
(D) $\frac{5}{7}$
(E) $\frac{2}{5}$
11. If n is an odd positive integer, what is the remainder when (n^2-1) is divided by 8?
(A) 1
(B) 0
(C) 5
(D) 4
(E) 6
12. What is the greatest prime factor of $(3^{30}-3^{27})$?
(A) 3
(B) 17
(C) 13
(D) 2
(E) 0
13. Which of the following is the last two digits of 7^{946} ?

- (A) 17
(B) 29
(C) 49
(D) 31
(E) 43
14. A clinic worked every day last week and had at least one patient everyday. If 10 patients went there by bus, what is the probability that at least 2 patients went to the clinic on the same day?
(A) 1
(B) $1/2$
(C) $1/3$
(D) $7/10$
(E) 无法计算
15. Which of the following integer can be expressed as the product of four different integers between -5 and 4 , inclusive?
(A) 64
(B) -100
(C) 75
(D) 56
(E) -120
16. The median of five people's (A, B, C, D and E) salary is 20000 and the range of them is no more than 50000, if the salary of A, B, C is 20000, 40000 and 50000 respectively, which of the following could be the mean of five people's salary?
(A) 21200
(B) 1900
(C) 42300
(D) 26000
(E) 315000
17. A rectangular block of goods is put in a rectangular box of 4 inches by 6 inches and 10 inches. There is a $\frac{1}{2}$ inch thick material wrapped around the block of goods to protect it and just fit in the box tightly, what is the volume of the block of goods?
(A) 240
(B) 320
(C) 360
(D) 288
(E) 135
18. The difference of the reciprocals of two consecutive positive integers could be?
(A) $7/30$
(B) $1/26$
(C) $3/20$
(D) $11/60$
(E) $1/72$
19. A list has 100 numbers, the expectation of the list is 10 and the standard deviation of the list is 4, then which pair of the integers would make the integers would make the standard deviation of the list decrease when the integers are put into the list?
(A) (0,20)
(B) (0,10)
(C) (10,10)
(D) $(-10,0)$
(E) $(-10, 10)$
20. If $a < b < c < 0$, then $\frac{a+b}{b+c}$ is
(A) less than 1
(B) greater than 1

- (C) less than
 (D) greater than 0, but less than 1
 (E) greater than $\frac{a}{c}$

21. For all integers a and b , the operation $@$ is defined as $a @ b = (-a + b)(b + a)$. If $a = 2$ and $b = 5$, then $b @ a =$

- (A) 25
 (B) -21
 (C) 7
 (D) 21
 (E) 25

22. How many numbers do the sets S and T have in common?

- (1) S is a set of 10 numbers.
 (2) T is a set of 100 numbers.

23. In the figure below, if line λ_1 is parallel to line λ_2 , what is the value of x ?

- (1) $y = 50$
 (2) $z = 130$

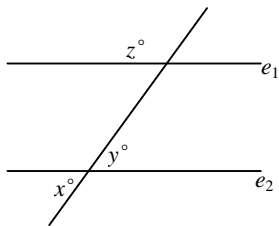


图 10.5

24. If a and b are positive integers, what is the value of $a + b$?

- (1) $\frac{a}{b} = \frac{5}{8}$
 (2) The greatest common divisor of a and b is 1.

25. If Pat saved \$600 of his earnings last month, how much did Pat earn last

month?

(1) Pat spent $\frac{1}{2}$ of his earnings last month for living expenses and saved $\frac{1}{3}$ of the remainder.

(2) Of his earnings last month, Pat paid twice as much in taxes as he saved.

26. The purchase price of Beth's new car, including the sales tax, is \$8,000. If she finances the car, making a down payment of \$2,000 and paying off the rest in equal monthly installments, what will be the total cost of the car, including the sales tax and financing?

- (1) The installments are to be \$200 per month.
 (2) The installments will extend over a period of exactly 3 years.

27. If $y \neq 0$, is $\frac{x}{y} = \frac{1}{2}$?

- (1) x is 50 percent of y .
 (2) $0.1x = 0.05y$

28. If n is an integer, is n even?

- (1) $n^2 - 1$ is an odd integer.
 (2) $3n + 4$ is an even integer.

29. If x , y , p , and q are positive, is $x \geq y$?

- (1) $\frac{px}{qy} = \frac{p}{q}$
 (2) $xy = p$

30. If p_1 and p_2 are the populations and r_1 and r_2 are the numbers of representatives of District 1 and District 2, respectively, the ratio of the population to the number of representatives is greater for which of

the two districts?

- (1) $p_1 > p_2$
 (2) $r_2 > r_1$

31. What digit does t represent in the decimal $0.t73$?

- (1) $t < 5$
 (2) $0.t73 < \frac{1}{t}$

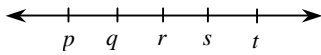


图 10.6

32. On the number line above, p , q , r , s , and t are five consecutive even integers in increasing order. What is the average (arithmetic mean) of these five integers?

- (1) $q + s = 24$
 (2) The average (arithmetic mean) of q and r is 11.

33. What is the length in meters of a certain rectangular garden?

- (1) The length of the garden is 6 meters more than twice the width.
 (2) The length of the garden is 4 times the width.

34. If $x < y$, is $x^2 < y^2$?

- (1) $y > 0$
 (2) $x > 0$

35. If Fran jumps straight up off the floor and lands on her feet T seconds later, her feet will reach a maximum height of $1.22T^2$ meters above the floor. On one such jump, was Fran off the floor for less than 1 second?

- (1) On her jump Fran's feet reached a maximum height of 1 meter above the floor.
 (2) On her jump Fran spent more than $\frac{1}{4}$ second ascending.

36. If $[x]$ denotes the greatest integer less than or equal to x , is $[x] = 0$?

- (1) $5x + 1 = 3 + 2x$
 (2) $0 < x < 1$

37. During a 6-day local trade show, the least number of people registered in a single day was 80. Was the average (arithmetic mean) number of people registered per day for the 6 days greater than 90?

- (1) For the 4 days with the greatest number of people registered, the average (arithmetic mean) number registered per day was 100.
 (2) For the 3 days with the smallest number of people registered, the average (arithmetic mean) number registered per day was 85.

参考答案

SECTION 1

1~5	CABCC
6~10	DADDC
11~15	DEDDA
16~20	BECDC
21~25	ADDEB
26~30	EBCEB
31~35	DAECD
36~37	CE

SECTION 2

1~5	ADEDC
6~10	DDACE
11~15	CCECB
16~20	CCACD
21~25	CBEDE
26~30	CEEEA
31~35	DADDB
36~37	EA

SECTION 3

1~5	CDABB
6~10	CEDDA
11~15	BCCEE
16~20	DEECB
21~25	BEDCA
26~30	CDDAC
31~35	EDCBA
36~37	DA

第 12 章 补充题集

Q1:

A certain roller coaster has 3 cars, and a passenger is equally likely to ride in any 1 of the 3 cars each time that passenger rides the roller coaster. If a certain passenger is to ride the roller coaster 3 times, what is the probability that the passenger will ride in each of the 3 cars?

- A 0
- B $1/9$
- C $2/9$
- D $1/3$
- E 1

Answer: -----

Q2:

In 1997 there were 300 female employees at Company C. If the number of female employees at Company C increased by 60 percent from 1977 to 1987, by what percent did the number of female employees at Company C increase from 1987 to 1997?

- (1) From 1977 to 1997 the number of female employees increased by 200 percent at Company C.
- (2) In 1977 there were 100 female employees at Company C.

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q3:

A gardener is going to plant 2 red rosebushes and 2 white rosebushes. If the gardener is to select each of the bushes at random, one at a time, and plant them in a row, what is the probability that the 2 rosebushes in the middle of the row will be the red rosebushes?

- A 1/12
- B 1/6
- C 1/5
- D 1/3
- E 1/2

Answer: -----

Q4:

If $1050 - 74$ is written as an integer in base 10 notation, what is the sum of the digits in that integer?

- A 424
- B 433
- C 440
- D 449
- E 467

Answer:

Q5:

If w and c are integers, is $w > 0$?

- (1) $w + c > 50$
- (2) $c > 48$

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q6:

A photographer will arrange 6 people of 6 different heights for photograph by placing them in two rows of three so that each person in the first row is standing in front of someone in the second row. The heights of the people within each row must increase from left to right, and each person in the second row must be taller than the person standing in front of him or

her. How many such arrangements of the 6 people are possible?

- A 5
- B 6
- C 9
- D 24
- E 36

Answer:

Q7:

Running at their respective constant rates, machine X takes 2 days longer to produce w widgets than machine Y. At these rates, if the two machines together produce $\frac{5}{4}w$ widgets in 3 days, how many days would it take machine X alone to produce $2w$ widgets?

- A 4
- B 6
- C 8
- D 10
- E 12

Answer:

Q8:

Joanna bought only \$0.15 stamps and \$0.29 stamps. How many \$0.15 stamps did she buy?

- (1) She bought \$4.40 worth of stamps.
- (2) She bought an equal number of \$0.15 stamps and \$0.29 stamps.

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q9:

A positive integer n is said to be “prime-saturated” if the product of all the different positive prime factors of n is less than the square root of n . What is the greatest two-digit prime-saturated integer?

- A 99
- B 98
- C 97
- D 96
- E 95

Answer:

Q10:

For a convention, a hotel charges a daily room rate of \$120 for 1 person and x dollars for each additional person. What is the charge for each additional person?

- (1) The daily cost per person for 4 people sharing the cost of a room equally is \$45.
- (2) The daily cost per person for 2 people sharing the cost of a room equally is \$25 more than the corresponding cost for 4 people.

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q11:

What is the median number of employees assigned per project for the projects at Company Z?

- (1) 25 percent of the projects at Company Z have 4 or more employees assigned to each project.
- (2) 35 percent of the projects at Company Z have 2 or fewer employees assigned to each project.

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q12:

A jar contains 16 marbles, of which 4 are red, 3 are blue, and the rest are yellow. If 2 marbles are to be selected at random from the jar, one at a time without being replaced, what is the probability that the first marble selected will be red and the second marble selected will be blue?

- A $3/64$
- B $1/20$
- C $1/16$
- D $1/12$
- E $1/8$

Answer:

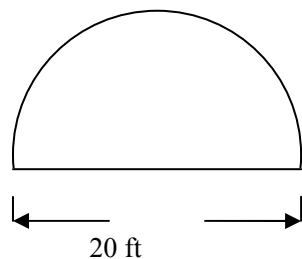
Q13:

When positive integer x is divided by positive integer y , the remainder is 9. If $x/y = 96.12$, what is the value of y ?

- A 96
- B 75
- C 48
- D 25
- E 12

Answer:

Q14:



The figure above shows the dimensions of a semicircular cross section of a one-way tunnel. The single traffic lane is 12 feet wide and is equidistant from the sides of the tunnel. If vehicles must clear the top of the tunnel by at least $\frac{1}{2}$ foot when they are inside the traffic lane, what should be the limit on the height of vehicles that are allowed to use the tunnel?

- A $5\frac{1}{2}$ ft
- B $7\frac{1}{2}$ ft

- C $8\frac{1}{2}$ ft
 D $9\frac{1}{2}$ ft
 E 10 ft

Answer:

Q15:

The operation \otimes is defined for all nonzero numbers a and b by $a \otimes b = a/b - b/a$. If x and y are nonzero numbers, which of the following statements must be true?

$x \otimes xy = x(1 \otimes y)$
 $x \otimes y = -(y \otimes x)$
 $1/x \otimes 1/y = y \otimes x$

- A. I only
 B. II only
 C. III only
 D. I and II
 E. II and III

Answer:

Q16:

If a and b are positive, is $(a-1 + b-1)^{-1}$ less than $(a-1)(b-1)^{-1}$?

- (1) $a = 2b$
 (2) $a + b > 1$

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
 B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
 C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
 D. EACH statement ALONE is sufficient.
 E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q17:

On a certain transatlantic crossing, 20 percent of a ship's passengers held round-trip tickets and also took their cars aboard the ship. If 60 percent of the passengers with round-trip tickets did not take their cars aboard the ship, what percent of the ship's passengers held

round-trip tickets?

- A 33 1/3%
- B 40%
- C 50%
- D 60%
- E 66 2/3%

Answer:

Q18:

A construction company was paid a total of \$500,000 for a construction project. The company's only costs for the project were for labor and materials. Was the company's profit for the project greater than \$150,000?

- (1) The company's total cost was three times its cost for materials.
- (2) The company's profit was greater than its cost for labor.

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

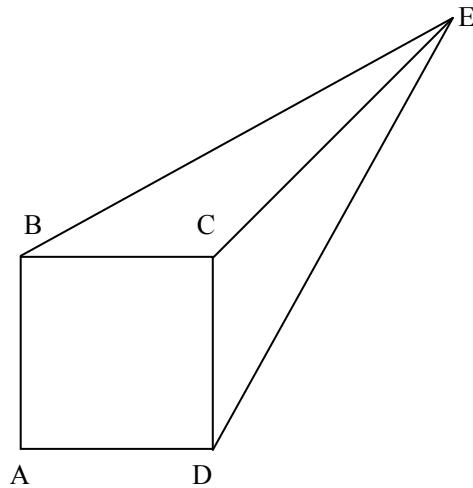
Q19:

There are 8 books on a shelf, of which 2 are paperbacks and 6 are hardbacks. How many possible selections of 4 books from this self include at least one paperback?

- A 40
- B 45
- C 50
- D 55
- E 60

Answer:

Q20:



In the figure, each side of square ABCD has length 1, the length of line segment CE is 1, and the length of line segment BE is equal to the length of line segment DE. What is the area of the triangular region BCE?

- A $1/3$
- B $(\sqrt{2})/4$
- C $1/2$
- D $(\sqrt{2})/2$
- E $3/4$

Answer:

Q21:

There are 8 magazines lying on a table; 4 are fashion magazines and the other 4 are sports magazines. If 3 magazines are to be selected at random from the 8 magazines, what is the probability that at least one of the fashion magazines will be selected?

- A $1/2$
- B $2/3$
- C $32/35$
- D $11/12$
- E $13/14$

Answer:

Q22:

What is the remainder when the two digit, positive integer x is divided by 3?

- (1) The sum of the digits of x is 5.
(2) The remainder when x is divided by 9 is 5.

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q23:

If $(y+3)(y-1) - (y-2)(y-1) = r(y-1)$, what is the value of y ?

- (1) $r^2 = 25$
(2) $r = 5$

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q24:

If n and k are positive integers, is n/k an even integer?

- (1) n is divisible by 8.
(2) k is divisible by 4.

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q25:

If n is a positive integer and r is the remainder when $(n-1)(n+1)$ is divided by 24, what is the value of r ?

- (1) 2 is not a factor of n .
(2) 3 is not a factor of n .

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q26:

Which of the following is equal to $[1/(\sqrt{3} - \sqrt{2})]^2$?

- A 1
B 5
C $\sqrt{6}$
D $5 - \sqrt{6}$
E $5 + 2\sqrt{6}$

Answer:

Q27:

If S is a set of ten consecutive integers, is the integer 5 in S ?

- (1) The integer -3 is in S .
(2) The integer 4 is in S .

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q28:

If Line k in the xy -plane has equation $y = mx + b$, where m and b are constants, what is the slope of k ?

- (1) k is parallel to the line with equation $y = (1-m)x + b + 1$.
(2) k intersects the line with equation $y = 2x + 3$ at the point $(2, 7)$.

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q29:

If M is the least common multiple of 90, 196, and 300, which of the following is NOT a factor of M ?

- A 600
B 700
C 900
D 2,100
E 4,900

Answer:

Q30:

What is the ratio of the average (arithmetic mean) height of students in class X to the average height of students in class Y ?

- (1) The average height of the students in class X is 120 centimeters.
(2) The average height of the students in class X and class Y combined is 126 centimeters.

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q31:

On Saturday morning, Malachi will begin a camping vacation and he will return home at the end of the first day on which it rains. If on the first three days of the vacation the probability of rain on each day is 0.2, what is the probability that Malachi will return home at the end of the day on the following Monday?

- A 0.008
- B 0.128
- C 0.488
- D 0.512
- E 0.640

Answer:

Q32:

If $x < 0$, then $\sqrt{(-x | x |)}$ is

- A $-x$
- B -1
- C 1
- D x
- E \sqrt{x}

Answer:

Q33:

If $x > 0.9$, which of the following could be the value of x ?

- A $\sqrt{0.81}$
- B $\sqrt{0.9}$
- C $(0.9)^2$
- D $(0.9)(0.99)$
- E $1 - \sqrt{0.01}$

Answer:

Q34:

If Antonio bought two half-gallon cartons of ice cream during a special sale, what percent of the total regular price of the two cartons did he save?

- (1) Antonio paid the regular price for the first carton and received the second carton for

half the regular price.

(2) The regular price of the ice cream Antonio bought was \$4.00 per half-gallon carton.

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q35:

If n is a positive integer, which of the following is a possible value of $|56 - 5n|$?

- A 7
- B 9
- C 12
- D 15
- E 20

Answer:

Q36:

If a committee of 3 people is to be selected from among 5 married couples so that the committee does not include two people who are married to each other, how many such committees are possible?

- A 20
- B 40
- C 50
- D 80
- E 120

Answer:

Q37

$$\sqrt{2\sqrt{63} + 2/(8+3\sqrt{7})} =$$

- A $8 + 3\sqrt{7}$
- B $4 + 3\sqrt{7}$
- C 8
- D 4
- E $\sqrt{7}$

Answer:

Q38:

The infinite sequence $a_1, a_2, \dots, a_n, \dots$ is such that $a_1 = 2$, $a_2 = -3$, $a_3 = 5$, $a_4 = -1$, and $a_n = a_{n-4}$ for $n > 4$. What is the sum of the first 97 terms of the sequence?

- A 72
- B 74
- C 75
- D 78
- E 80

Answer:

Q39:

What is the value of $|x + 7|$?

- (1) $|x + 3| = 14$
- (2) $(x + 2)^2 = 169$

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q40:

In the xy -plane, the point $(-2, -3)$ is the center of a circle. The point $(-2, 1)$ lies inside the circle and the point $(4, -3)$ lies outside the circle. If the radius r of the circle is an integer, then $r =$

- A 6
- B 5
- C 4
- D 3
- E 2

Answer:

Q41:

A driver completed the first 20 miles of a 40-mile trip at an average speed of 50 miles per hour. At what average speed must the driver complete the remaining 20 miles to achieve an average speed of 60 miles per hour for the entire 40-mile trip? (Assume that the driver did not make any stops during the 40-mile trip.)

- A 65 mph
- B 68 mph
- C 70 mph
- D 75 mph
- E 80 mph

Answer:

Q42:

Professor Vásquez gave a quiz to two classes. Was the range of scores for the first class equal to the range of scores for the second class?

- (1) In each class, the number of students taking the quiz was 26, and the lowest score in each class was 70.
- (2) In each class, the average (arithmetic mean) score on the quiz was 85.

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q43:

Of the families in City X in 1994, 40 percent owned a personal computer. The number of families in City X owning a computer in 1998 was 30 percent greater than it was in 1994, and the total number of families in City X was 4 percent greater in 1998 than it was in 1994. What percent of the families in City X owned a personal computer in 1998?

- A 50%
- B 52%
- C 56%
- D 70%
- E 74%

Answer:

Q44:

If b , c , and d are constants and $x^2 + bx + c = (x + d)^2$ for all values of x , what is the value of c ?

- (1) $d = 3$
- (2) $b = 6$

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q45:

If a , b , k , and m are positive integers, is ak a factor of bm ?

- (1) a is a factor of b .
- (2) $k \leq m$

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q46:

If the average (arithmetic mean) of four different numbers is 30, how many of the numbers are greater than 30?

- (1) None of the four numbers is greater than 60.
- (2) Two of the four numbers are 9 and 10, respectively.

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q47:

If n is the greatest positive integer for which $2n$ is a factor of $10!$, then $n =$

- A 2
- B 4
- C 6
- D 8
- E 10

Answer:

Q48:

Is $|x - y| > |x - z|$?

- (1) $|y| > |z|$
- (2) $x < 0$

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q49:

At a certain company, 25 percent of the employees are male and 50 percent of the employees are sales staff. What is the number of employees at this company?

- (1) Exactly 7 of the employees at the company are males who are sales staff.
- (2) There are 16 more female employees than male employees at the company.

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q50:

If $-4 < x < 7$ and $-6 < y < 3$, which of the following specifies all the possible values of xy ?

- A $-42 < xy < 21$
- B $-42 < xy < 24$
- C $-28 < xy < 18$
- D $-24 < xy < 21$
- E $-24 < xy < 24$

Answer:

Q51:

From a group of 3 boys and 3 girls, 4 children are to be randomly selected. What is the probability that equal numbers of boys and girls will be selected?

- A $\frac{1}{10}$
- B $\frac{4}{9}$
- C $\frac{1}{2}$
- D $\frac{3}{5}$
- E $\frac{2}{3}$

Answer:

Q52:

For any integer k greater than 1, the symbol k^* denotes the product of all the fractions of the

form $\frac{1}{t}$, where t is an integer between 1 and k , inclusive. What is the value of $\frac{5^*}{4^*}$?

- A 5
- B $\frac{5}{4}$
- C $\frac{4}{5}$
- D $\frac{1}{4}$

E $\frac{1}{5}$

Answer:

Q53:

Company S produces two kinds of stereos: basic and deluxe. Of the stereos produced by

Company S last month, $\frac{2}{3}$ were basic and the rest were deluxe. If it takes $\frac{7}{5}$ as many hours to produce a deluxe stereo as it does to produce a basic stereo, then the number of hours it

took to produce the deluxe stereos last month was what fraction of the total number of hours it took to produce all the stereos?

A $\frac{7}{17}$

B $\frac{14}{31}$

C $\frac{7}{15}$

D $\frac{17}{35}$

E $\frac{1}{2}$

Answer:

Q54:

On a map Town G is 10 centimeters due east of Town H and 8 centimeters due south of Town J. Which of the following is closest to the straight-line distance, in centimeters, between Town H and Town J on the map?

A 6

B 13

C 18

D 20

E 24

Answer:

Q55:

If an integer n is to be chosen at random from the integers 1 to 96, inclusive, what is the probability that $n(n + 1)(n + 2)$ will be divisible by 8?

A $\frac{1}{4}$

B $\frac{3}{8}$

C $\frac{1}{2}$

D $\frac{5}{8}$

E $\frac{3}{4}$

Answer:

Q56:

The annual rent collected by a corporation from a certain building was x percent more in 1998 than in 1997 and y percent less in 1999 than in 1998. Was the annual rent collected by the corporation from the building more in 1999 than in 1997?

(1) $x > y$

(2) $\frac{xy}{100} < x - y$

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q57:

The numbers x and y are three-digit positive integers, and $x + y$ is a four-digit integer. The tens digit of x equals 7 and the tens digit of y equals 5. If $x < y$, which of the following must be true?

I. The units digit of $x + y$ is greater than the units digit of either x or y .

- II. The tens digit of $x + y$ equals 2.
III. The hundreds digit of y is at least 5.

- A. II only
B. III only
C. I and II
D. I and III
E. II and III

Answer:

Q58:

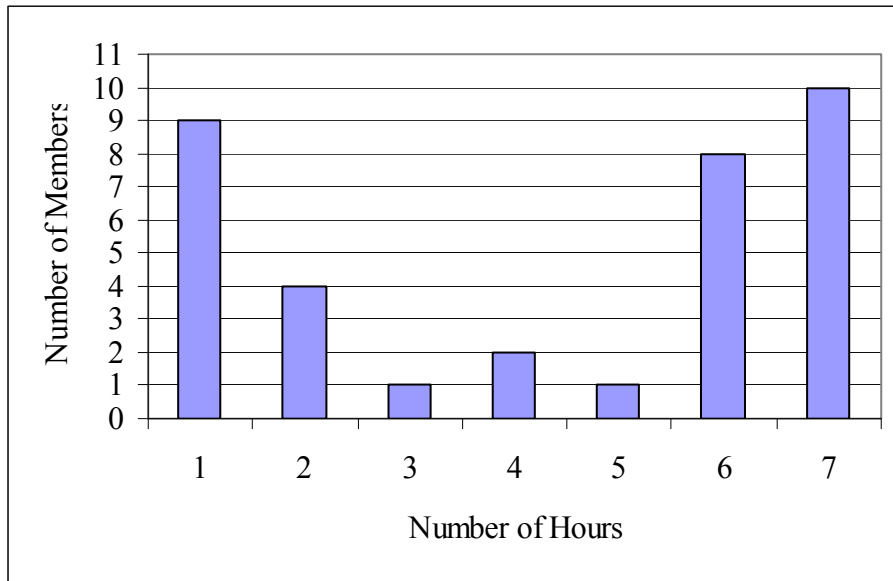
If n is an integer and $100 < n < 200$, what is the value of n ?

- (1) $\frac{n}{36}$ is an odd integer.
(2) $\frac{n}{45}$ is an even integer.

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q59:



Yesterday each of the 35 members of a certain task force spent some time working on project P. The graph shows the number of hours and the number of members who spent that number of hours working on project P yesterday. What was the median number of hours that the members of the task force spent working on project P yesterday?

- A 2
- B 3
- C 4
- D 5
- E 6

Answer:

Q60:

The average (arithmetic mean) of the 5 positive integers k , m , r , s , and t is 16, and $k < m < r < s < t$. If t is 40, what is the greatest possible value of the median of the 5 integers?

- A 16
- B 18
- C 19
- D 20
- E 22

Answer:

Q61:

	Favorable	Unfavorable	Not Sure
Candidate M	40	20	40
Candidate N	30	35	35

The table above shows the results of a survey of 100 voters each responded “favorable” or “unfavorable” or “not sure” when asked about their impressions of candidate M and of candidate N. What was the number of voters who responded “favorable” for both candidates?

- (1) The number of voters who did not respond “favorable” for either candidate was 40.
- (2) The number of voters who responded “unfavorable” for both candidates was 10.

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q62:

When tossed, a certain coin has equal probability of landing on either side. If the coin is tossed 3 times, what is the probability that it will land on the same side each time?

- A $\frac{1}{8}$
- B $\frac{1}{4}$
- C $\frac{1}{3}$
- D $\frac{3}{8}$
- E $\frac{1}{2}$

Answer:

Q63:

Is $|x| + |x - 1| = 1$?

- (1) $x \geq 0$
 (2) $x \leq 1$

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
 B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
 C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
 D. EACH statement ALONE is sufficient.
 E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

Q64:

If x , y , and k are positive numbers such that $\frac{x}{x+y}(10) + \frac{y}{x+y}(20) = k$ and if $x < y$, which of the following could be the value of k ?

- A 10
 B 12
 C 15
 D 18
 E 30

Answer:

Q65:

A certain quantity is measured on two different scales, the R-scale and the S-scale, that are related linearly. Measurements on the R-scale of 6 and 24 correspond to measurements on the S-scale of 30 and 60, respectively. What measurement on the R-scale corresponds to a measurement of 100 on the S-scale?

- A 20
 B 36
 C 48
 D 60
 E 84

Answer:

Q66:

If the sequence $x_1, x_2, x_3, \dots, x_n, \dots$ is such that $x_1 = 3$ and $x_{n+1} = 2x_n - 1$ for $n \geq 1$, then $x_{20} - x_{19} =$

- A 219
- B 220
- C 221
- D $220 - 1$
- E $221 - 1$

Answer:

Q67:

When positive integer x is divided by 5, the remainder is 3; and when x is divided by 7, the remainder is 4. When positive integer y is divided by 5, the remainder is 3; and when y is divided by 7, the remainder is 4. If $x > y$, which of the following must be a factor of $x - y$?

- A 12
- B 15
- C 20
- D 28
- E 35

Answer:

Q68:

One kilogram of a certain coffee blend consists of x kilogram of type I coffee and y kilogram of type II coffee. The cost of the blend is C dollars per kilogram, where $C = 6.5x + 8.5y$. Is $x < 0.8$?

- (1) $y > 0.15$
- (2) $C \geq 7.30$

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are NOT sufficient.

Answer:

参考答案

1-5 CDBCE	6-10 AEADD	11-15 CBBBE	16-20 BCCDB
21-25 EDEEC	26-30 EEAAE	31-35 BABAB	36-40 DDBCB
41-45 DEADC	46-50 CDEBB	51-55 DEABD	56-60 BBEB
61-65 ABCDC	66-68 AEB		

第四部分 附录

附录A: GMAT数学常见术语

一、代数部分

1. 数学运算

add, plus 加
subtract 减
difference 差
multiply, times 乘
product 积
divide 除
divisible 可被整除的
divided evenly 被整除
dividend 被除数, 红利
divisor 因子, 除数
quotient 商
remainder 余数
factorial 阶乘
power 乘方
radical sign, root sign 根号
round to 四舍五入
to the nearest 四舍五入

2. 集合

union 并集
proper subset 真子集
solution set 解集

3. 代数式、方程和不等式

algebraic term 代数项
like terms, similar terms 同类项
numerical coefficient 数字系数

literal coefficient 字母系数
inequality 不等式
triangle inequality 三角不等式
range 值域
original equation 原方程
equivalent equation 同解方程, 等价方程
linear equation 线性方程

4. 分数和小数

proper fraction 真分数
improper fraction 假分数
mixed number 带分数
vulgar fraction, common fraction 普通分数
simple fraction 简分数
complex fraction 繁分数
numerator 分子
denominator 分母
(least) common denominator (最小)公分母
quarter 四分之一
decimal fraction 纯小数
infinite decimal 无穷小数
recurring decimal 循环小数
tenths unit 十分位

5. 基本数学概念

arithmetic mean 算术平均值
weighted average 加权平均值
geometric mean 几何平均数
exponent 指数, 幂
base 乘幂的底数, 底边
cube 立方数, 立方体

square root 平方根
 cube root 立方根
 common logarithm 常用对数
 digit 数字
 constant 常数
 variable 变量
 inverse function 反函数
 complementary function 余函数
 linear 一次的, 线性的
 factorization 因式分解
 absolute value 绝对值
 round off 四舍五入

6. 有关数论

natural number 自然数
 positive number 正数
 negative number 负数
 odd integer, odd number 奇数
 even integer, even number 偶数
 integer, whole number 整数
 positive whole number 正整数
 negative whole number 负整数
 consecutive number 连续整数
 real number, 实数,
 rational number 有理数
 irrational(number) 无理数
 inverse 倒数
 composite number 合数
 prime number 质数
 reciprocal 倒数
 common divisor 公约数
 multiple 倍数
 (least)common multiple (最小)公倍数

(prime) factor (质)因子
 common factor 公因子
 ordinary scale, decimal scale 十进制
 nonnegative 非负的
 tens 十位
 units 个位
 mode 众数
 median 中数
 common ratio 公比

7. 数列

arithmetic progression(sequence) 等差数列
 geometric progression(sequence) 等比数列

8. 其他

approximate 近似
 (anti)clockwise (逆) 顺时针方向
 cardinal 基数
 ordinal 序数
 direct proportion 正比
 distinct 不同的
 estimation 估计, 近似
 parentheses 括号
 proportion 比例
 permutation 排列
 combination 组合
 table 表格
 trigonometric function 三角函数
 unit 单位,位

二、几何部分

1. 角

alternate angle 内错角

corresponding angle 同位角

vertical angle 对顶角

central angle 圆心角

interior angle 内角

exterior angle 外角

supplementary angles 补角

complementary angle 余角

adjacent angle 邻角

acute angle 锐角

obtuse angle 钝角

right angle 直角

round angle 周角

straight angle 平角

rectangular solid 长方体

included angle 夹角 **2. 三角形**

equilateral triangle 等边三角形

scalene triangle 不等边三角形

isosceles triangle 等腰三角形

right triangle 直角三角形

oblique 斜三角形

inscribed triangle 内接三角形

3. 有关收敛的平面图形, 除三角形外

semicircle 半圆

concentric circles 同心圆

quadrilateral 四边形

pentagon 五边形

hexagon 六边形

heptagon 七边形

octagon 八边形

nonagon 九边形

decagon 十边形

polygon 多边形

parallelogram 平行四边形

equilateral 等边三角形

plane 平面

square 正方形, 平方

rectangle 长方形

regular polygon 正多边形

rhombus 菱形

trapezoid 梯形

4. 其他平面图形

arc 弧

line, straight line 直线

line segment 线段

parallel lines 平行线

segment of a circle 弧形

5. 立体图形

cube 立方体, 立方数

rectangular solid 长方体

regular solid/regular polyhedron 正多面体

circular cylinder 圆柱体

cone 圆锥

sphere 球体

solid 立体的

6. 图形上的附属物

altitude 高

depth 深度

side 边长

circumference, perimeter 周长

radian 弧度

surface area 表面积

volume 体积

arm 直角三角形的股

cross section 横截面

center of a circle 圆心

chord 弦

radius 半径
 angle bisector 角平分线
 diagonal 对角线
 diameter 直径
 edge 棱
 face of a solid 立体的面
 hypotenuse 斜边
 included side 夹边
 leg 三角形的直角边
 median of a triangle 三角形的中线
 base 底边, 基数
 opposite 直角三角形中的对边
 midpoint 中点
 endpoint 端点
 vertex (复数形式 vertices) 顶点
 tangent 切线的
 transversal 截线
 intercept 截距

7. 坐标

coordinate system 坐标系
 rectangular coordinate 直角坐标系
 origin 原点
 abscissa 横坐标
 ordinate 纵坐标
 number line 数轴
 quadrant 象限
 slope 斜率
 complex plane 复平面

8. 其他

plane geometry 平面几何
 trigonometry 三角学
 bisect 平分

circumscribe 外切
 inscribe 内切
 intersect 相交
 perpendicular 垂直
 pythagorean theorem 勾股定理
 congruent 全等的
 multilateral 多边的

三、其他

intercalary year(leap year) 闰年(366 天)
 common year 平年(365 天)
 depreciation 折旧
 down payment 直接付款
 discount 打折
 margin 利润
 profit 利润
 interest 利息
 simple interest 单利
 compounded interest 复利
 dividend 红利
 decrease to 减少到
 decrease by 减少了
 increase to 增加到
 increase by 增加了
 denote 表示
 list price 标价
 markup 涨价
 per capita 每人
 ratio 比率
 retail price 零售价
 tie 打平

附录B: GMAT数学常见公式

一、算术

$$1. P_n^m = \frac{n!}{(n-m)!} = m(m-1)(m-2)\dots(m-n+1)$$

$$2. C_n^m = \frac{n!}{m!(n-m)!} \\ = \frac{n(n-1)(n-2)\dots(n-m+1)}{1 \times 2 \times 3 \dots \times m}$$

$$3. N \text{ 的阶乘公式: } N! = 1 \times 2 \times 3 \times \dots \times (N-2) \times (N-1) \times N \text{ 且规定 } 0! = 1, 1! = 1$$

二、代数:

$$1. \text{一元二次方程 } ax^2 + bx + c = 0 (a \neq 0) \text{ 的求根公式是:}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} (b^2 - 4ac \geq 0)$$

$$2. a_n = a_1 + (n-1)d \\ S_n = \frac{n(a_1 + a_n)}{2} = na_1 + \frac{n(n-1)d}{2}$$

- 当 n 为偶数时, M 为中间两项的算术

$$\text{平均值 } M = \frac{\frac{a_n}{2} + \frac{a_{n+1}}{2}}{2}$$

当 n 为奇数时, M 为中间项 $M = a_{\frac{n+1}{2}}$

$$3. a_n = a_1 q^{n-1}$$

$$S_n = \frac{a_1(1-q^n)}{1-q} (q \neq 1), \text{ 或}$$

$$S_n = \frac{a_1 - a_n q}{1-q} (q \neq 1)$$

- 当 n 为偶数时, M 为中间两项的几何平

$$\text{均数: } M = \sqrt{\frac{a_n}{2} \times \frac{a_{n+1}}{2}}$$

- 当 n 为奇数时, M 为中间项: $M = a_{\frac{n+1}{2}}$

$$4. (1) a^m \cdot a^n = a^{m+n}$$

$$(2) a^m \div a^n = a^{m-n}$$

$$(3) (a^m)^n = a^{mn}$$

$$(4) a^{-m} = \frac{1}{a^m} (m > 0, a \neq 0)$$

$$(5) a_n^m = \sqrt[n]{a^m} (\frac{m}{n} \text{ 为最简分数, 当 } n \text{ 为正}$$

偶数时, a^m 必为非负数)

$$(6) (a \cdot b)^m = a^m \cdot b^m$$

$$(7) \frac{a^x}{b^x} = \left[\frac{a}{b} \right]^x$$

$$(8) a^0 = 1, (a \neq 0)$$

$$5. (1) a^2 - b^2 = (a+b)(a-b)$$

$$(2) a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$(3) a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$(4) a^2 + 2ab + b^2 = (a+b)^2$$

$$(5) a^2 - 2ab + b^2 = (a-b)^2$$

$$(6) a^3 + 3a^2b + 3ab^2 + b^3 = (a+b)^3$$

$$(7) a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3$$

$$(8) a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \\ = (a+b+c)^2$$

三、几何

1. 平面图形的周长和面积:

图形	周长	面积
Triangle	三边之和	(底×高)/2
Square	边长×4	边长的平方
Rectangle	(长+宽)×2	长×宽
Parallelogram	(长+宽)×2	底×高
Trapezoid	四边之和	(上底+下底)×高/2
Rhombus	边长×4	两条对角线之积的1/2
Circle	$2\pi r = \pi d$	πr 的平方

2. 立体图形的表面积和体积

图形: Rectangular Prism

体积: 长×宽×高

表面积: $2(\text{长} \times \text{宽} + \text{长} \times \text{高} + \text{宽} \times \text{高})$

图形: Cube

体积: 棱长的立方

表面积: $6 \times \text{棱长} \times \text{棱长}$

图形: Right Circular Cylinder

体积: πr 的平方× h

表面积: $2\pi rh$ (侧)+ $2\pi r^2$ (底)

图形: Sphere

体积: $4\pi r$ 的三次方/3

表面积: $4\pi r$ 的平方

图形: Right Circular Cone

体积: r 的平方× $h/3$

表面积: $lr/2$ (l 为母线)

3. 多边形内角和= $(n-2) \times 180$

4. 任意两点间的距离

$$P_1P_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

四、其他

华氏温度与摄氏温度的换算 $(F-32) \times 5/9 = ^\circ\text{C}$

附录C: 常见度量衡的转换

一、Linear Measure 长度

1 inch 英寸=25.4 millimeters 毫米

1 foot 英尺=12 inches 英寸=0.3048 meter 米

1 yard 码=3 feet 英尺=0.9144 meter 米

1 (statute) mile 英里=1760 yards 码=1.609 kilometers 千米

二、Square Measure 面积

1 square inch 平方英寸=6.45 sq. centimeters 平方厘米

1 square foot 平方英尺=144 sq. in. 平方英寸=9.29 sq. decimeters 平方分米

1 square yard 平方码=9 sq. ft. 平方英尺=0.836 sq. meter 平方米

三、Cubic Measure 体积

1 cubic inch 立方英寸=16.4 cu. centimeters 立方厘米

1 cubic foot 立方英尺=1728 cu. in. 立方英寸=0.0283 cu. meter 立方米

1 cubic yard 立方码=27 cu. ft. 立方英尺=0.765 cu. meter 立方米

四、固体容积 (美制)

1 pint 品脱=33.60 cu. in. 立方英寸=0.550 liter 升
1 quart 夸脱=2 pints 品脱=1.101 liters 升

五、液体容积 (美制)

1 pint 品脱=16 fluid oz. 液量盎司 =28.88 cu. in. 立方英寸=0.473 liter 升
1 quart 夸脱=2 pints 品脱=0.946 liter 升

1 gallon 加伦=4quarts 夸脱=3.785 liters 升

六、Weight 重量

1 dram 打兰=1.772 grams 克
1 ounce 盎司=16drams 打兰=28.35grams 克
1 pound 磅=16 ounces 盎司=7000 grains 谷=0.4536 kilogram 千克
1 short ton 短吨(美吨)=2000 pounds 磅=0.907 tonne 公吨